Structural Complex Systems and Their Analysis Problems

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Abstract:
The article discusses the analysis of structurally complex systems whose structure can be represented as a graph. A simple example of reducing a complex system to a recurrent form is given.

Keywords: system, structure, function, model, graph.

A vast number of parts and complicated interactions characterize modern systems. There are numerous functional designations for such systems. Some systems are built to handle difficult, perhaps life-threatening jobs.

It is especially important to analyze the systems under development during the design phase when constructing huge systems with a complicated structure and a high number of elements. This will help to prevent project errors in the future. A mathematical model of the system to be produced is created for this purpose.To make the system modeling and analysis process easier, it is assumed that the system pieces are self-contained, have no memory[1], and are not recoverable [2-15].

Models of systems with a simple daisy-chain or parallel structure are uncomplicated. Such systems fulfil their function F(X), provided that the system elements x_i,

$$F(X) = f(x_1) \wedge f(x_2) \wedge f(x_3) \dots \wedge f(x_n) \quad (1)$$

here n – the number of elements in the system under study, $f(x_i)$ – function of the *i*-th element, or in short

$$F(X) = \wedge f(x_i) \quad (2)$$

here $\wedge$ can be $\prod$, $\sum$ or alternate, depending on the nature of the system under investigation.

When analyzing the reliability of a system with a sequential structure, function (2) has the form

$$F(X)=\prod_{i=1}^{n} f(x_i) \quad (3)$$

For systems with a parallel structure

$$F(X)=1-\prod_{i=1}^{n}(1-f(x_i)) \quad (4)$$

In plurality systems of the "k" of "m" type, a system is considered operable when at least "k" of its elements are operable.

Denote the set of system elements by N, the number of operable elements by n, then

F(X)= 1 with n≥k where k is the minimum number of elements required to make the system work. (5)

With homogeneous system elements

$F(X)=\prod_{i=1}^{n} f(x_i)$,here $x_i \in N$andn$\geq$ к. (6)

Suppose that the set of elements of the system N can be split into j subsets.For each subset a different condition "k of n" is defined. Under these conditions, (5) will take the form

$F(X)=\prod_{r=1}^{j} \prod_{i=1}^{kr} f(x_{i,r})$,(7)

Here $x_{i,r}$- i-element of r-thsubset.

Most problems in analyzing systems with a complex structure are simplified by converting the structure of the system to an ordered, recurrent form [7-9]. Most authors [1,2,3] consider the bridging structure as an example

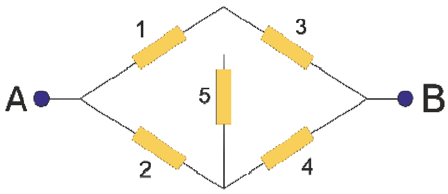

Fig.1

hereA – system input, B – system output.

For simplicity, replace the elements of the system with graph edges and their connections with nodes.

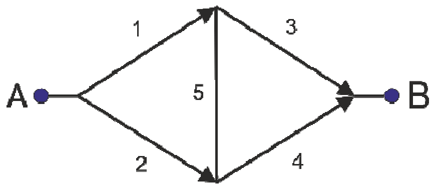

Fig.2

The system is considered to be operational if there is at least one path between the input of system A and the output of system B. In several cases [1,2,3], the authors provide minimum path search techniques for translating a structural model of complicated systems into a parallel-sequential one. The minimum path connects the fewest number of system elements necessary to assure the system's operability.

For the example given, the number of minimum paths is 4:

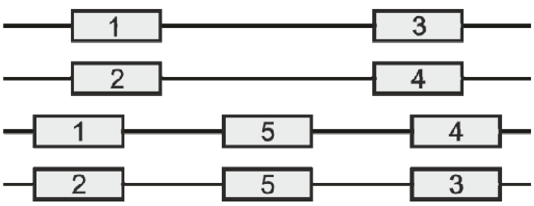

Fig.3

It is known [1,7,8,10-15] ,that the calculated value of F(X) by the minimum path method gives an estimate of its maximum value F (X).As the number of elements in the system increases, the process of finding minimum paths increases. In the study of structurally complex systems (SCS) using various methods, the task arises of adequately representing systems and their properties by models, and of obtaining estimates with the necessary accuracy [2]. The complexity of a system model grows sometimes exponentially as the number of elements increases. The latter forces resort to various techniques with a loss of information and accuracy of analysis [6,8]. Such systems' analytical models become clumsy and difficult to decipher. The application of graph theory can be made practical by using a model in the form of a graph. A graph is a collection of edges (arcs) and vertices connected by rules[2,3]. Most systems' structure may be easily represented as a graph. The majority of modern systems have structures like this. Communication networks, transportation networks, energy networks, and computer networks are all examples of networks.

The concept of system operability and its failure depends on the problem statement for the system under study and its purpose, which are taken into account when building the model.

Assume that the system elements and their links can be in a working state (x_i=1)or in a state of failure(x_i=0). System element failures are independent and cannot be recovered. For analyzing the SCS's reliability, most publications recommend using the minimum path and minimum cross-section (section) methods [1-4]. This kind of inquiry can be used to determine the maximum and minimum values of the system dependability indicator. To approximate the reliability of a system to its genuine value, many decomposition approaches are applied [2,5]. As an example, most authors use a bridge strategy (system).

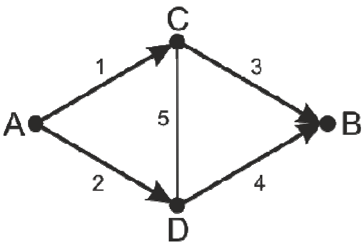

Fig.4

The authors [1,2] suggest a method of singular element decomposition based on a Boolean function decomposition with respect to its variables. Effectively identifying the element (edge) with respect to which to do the decomposition becomes a very challenging operation in more complicated systems with a large number of elements.

Consider using the widely used bridging strategy to arrange the system model as a graph to a recursive form.

Step 1: Replace element 5 with two differently oriented edges:

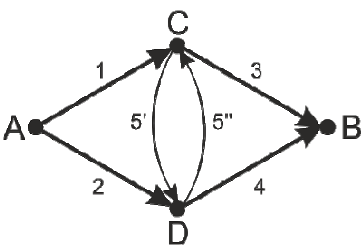

Fig.5

Step 2. Divide vertices C and D into two verticesC1, C2 and D1,D2, respectively.:

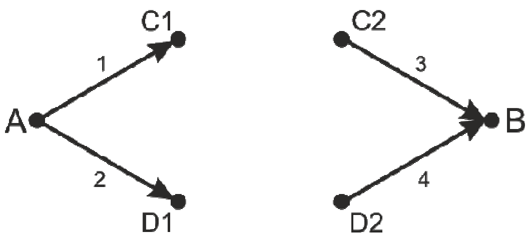

Fig.6

Step 3: Add dummy edges 6 and 7 connecting split edges, respectively:

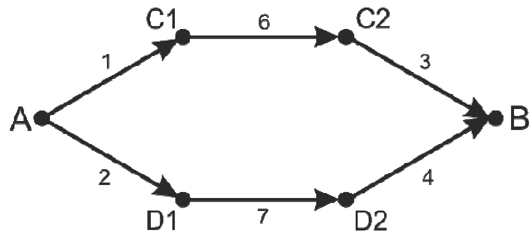

Fig.7

Step 4. Connect vertex C1 to vertex D1 with edge 5', vertex D1 to vertex C2 with edge 5’.

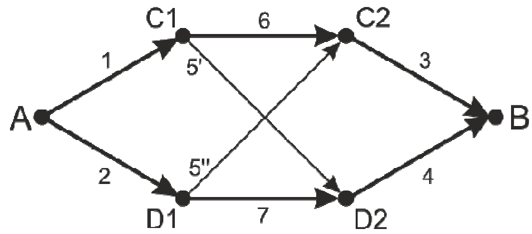

Fig.8

Step5. Assign edge 5' and 5" to edge 5. The added edges are assigned values that should not change the desired system value. In reliability tasks:$P_6 = P_7 = 1, P_{5'} = P_{5''} = P_5$.

The result is a graph whose edges are directed strictly from system input A to its output B.

Consider an example of calculating the reliability index of the system in question. Decompose the resulting system with a recurrence structure into subsystems. In the first subsystem, let's include the elements of the system connected to input A - edges 1 and 2. For edges 1 and 2 of subsystem 1, the input signal is the signal at the input of system A.

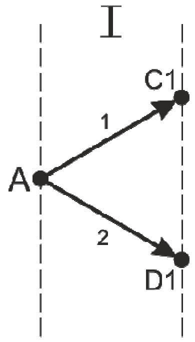

Fig.9

The second subsystem includes edges associated with elements in the first subsystem:

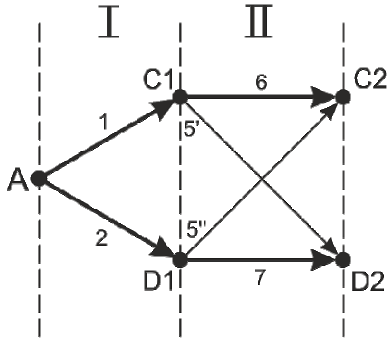

Fig.10

The inputs for edges 6, 5’, 5’’,7 are the outputs of the first subsystem C1 and D1.The output of the second subsystem is nodes C2 and D2. The third subsystem is made up of edges connected to elements in the second subsystem:

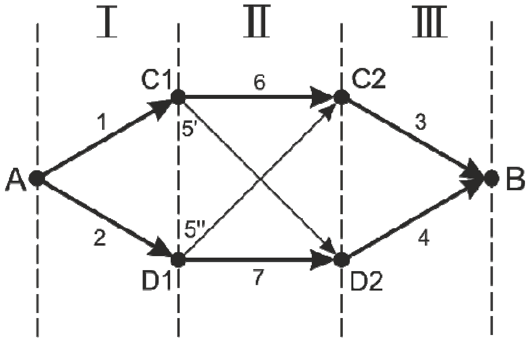

Fig.11

Accordingly, the input signal for the elements of the third subsystem are the output signals of the second subsystem. The output of this subsystem is the desired result.Assume that the failures of the elements of the system are independent, for the i-th element P_i=1 if it is operational and P_i=0 if it fails. The first subsystem has two outputs. We denote the state of communication between input A and node C1 and D1 by 1, and the state of no communication by 0. The outputs of the first subsystem can be in four states – S11, S12, S13, S14.

Make a table of states for the first subsystem

States	Vertex	
	C1	D1
S11	1	1
S12	1	0
S13	0	1
S14	0	0

The probabilities of these states can be calculated using known methods:

$P(S11)=P_1*P_2$

$P(S12)=P_1*(1-P_2)$

$P(S13)=(1-P_1) *P_2$

$P(S14)= (1 - P_1)*(1 - P_2)$

The second subsystem also has two outputs C2 and D2 and four states- S21, S22, S23, S24:

States	Vertex	
	C2	D2
S21	1	1
S22	1	0
S23	0	1
S24	0	0

Let's calculate the probabilities of these states taking into account $P_6 =P_7 =1$

$P(S21)=(1-(1-P(S11)*(1-P(S12) *P_{5'})*(1-P(S13) *P_{5''}))$

$P(S22)= (1-(1-P(S11))*(1-(PS12))*(1-P(S13)*P_{5''}))$

$P(S23)=(1 - (1 - P(S11)) * (1 - P(S12) * P_{5'}) * P(S13))$

Further, the third subsystem has only one output, respectively two states - S31, S32:

States	Vertex	
	B	
S31	1	
S32	0	

Calculate the probability of event S31:

$P(S31) = (1-P(S21) * (1-P_3) * (1-P_4)) * (1-(1-P(S22) * P_3) * (1-P(S23) * P_4))$

The calculated S31 state value is the required system reliability indicator.

By artificially reducing the structure of the system to a recurrent form, it can be broken down into successively connected subsystems. A simple algorithm can then be applied to such systems to calculate the reliability index of the system.

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