

The nonsmooth optimal control problem for ensemble of trajectories of dynamic system under conditions of indeterminacy

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Abstract: In the paper we consider the one model of dynamic system under conditions of indeterminacy – linear controllable differential inclusions. For the informational model of the control system the minimax control problem for ensemble trajectories is researched. This control problem is study with a methods nonsmooth and multi-value analysis. The necessary and sufficient conditions of optimality are obtained.

Keywords: differential inclusion, control system, informational model, nonsmooth functional, minimax problem, conditions of optimality.

1. Introduction

Effective control of complex technical objects and processes is impossible without taking into account factors such as incomplete information on external parameters and inaccuracy of the initial a priori data of the system. The control problems in conditions of limited information of various types lead to the so-called information models of control systems.

Known methods for studying a information models of control systems are developed taking into account the degree of limited information regarding external parameters and the initial state of the system. In applied problems, a more common situation is when a priori data on the initial state of the system and the parameters of external influences are minimal, i.e. there are no statistical descriptions of them, and information is limited to setting only the set of possible values of unknown parameters. In such cases, we can say that the consideration model represents a control system in conditions of uncertainty. For control systems under conditions of uncertainty, the properties of the ensemble of trajectories, methods for estimating the reachability set and forecasting the phase state of the system and others are of great interest.

Many applied problems encountered in economic planning and production organization, in the design of technical devices and process control in other areas, lead to nonsmooth optimization problems [1–3]. In these problems often appear are nonsmooth objective functions such as maximum or minimum [3]. Models of decision making under conditions of incomplete information on the initial data and external parameters of the system lead to optimization problems with such criteria [1]. In studies of such models control system, methods of nonsmooth analysis and the theory of differential inclusions are of great importance. Differential inclusions with control parameters are used as an effective mathematical apparatus [4–6].

The theory of differential inclusions in close connection with the theory of optimal control is developing in various directions [5–7]. One of the developing directions in the theory of differential inclusions and their applications is differential inclusions with control and other external parameters. Among the urgent tasks for such systems, it is worth noting such tasks as the dependence of the set of trajectories on the parameters and initial data, the controllability conditions of the ensemble of trajectories, and the optimization of the control of the ensemble of trajectories according to given criteria [8–11].

2. The object of study and methods

We consider differential inclusion of the form

$$\frac{dx}{dt} \in A(t)x + B(t, u), \quad (1)$$

where \mathcal{X} is the state vector, \mathbf{u} is the control vector with values from the $U \subset R^m$, $A(t)$ is the $n \times n$ matrix, $B(t, u)$ is the nonempty compact from space R^n . This class of differential inclusions can be used as a mathematical model of control objects, when information about external influences is incomplete. In fact, let us consider a mathematical model of the control object of the form

$$\frac{dx}{dt} = A(t)x + b(t, u, w), \quad (2)$$

where $w = w(t)$ is an uncontrolled external influence, the information about it is limited in the form of a condition

$w(t) \in W(t)$, $W(t)$ is a given compact set from the space of values of parameter W . Assuming measurability in t , continuous in (u, w) of the function $b(t, u, w)$ and measurability of function $w = w(t)$, based on the Lemma of Filippov-Kasten [13, p.179] can be seen that the control system (2) is equivalent to the controlled differential inclusion (1) with right-hand side $B(t, u) = b(t, u, W(t))$.

We consider a control problem for ensemble of trajectories of differential inclusion (1). We will assume that:

1) the elements of the matrices $A(t)$ are summable on a interval $T = [t_0, t_1]$;

2) the multi-valued map $(t, u) \rightarrow B(t, u)$ is measurable in $t \in T$ and continuous in $u \in U$, with

$$\text{Sup}_{\gamma \in b(t, u)} \|\gamma\| \leq \beta(t), \quad \forall (t, u) \in T \times V, \text{ where } \beta(t) \text{ is the summable function on } T.$$

By an admissible control for system (1) we mean a measurable bounded m -vector function $u = u(t)$ defined on a interval $T = [t_0, t_1]$ of time. Denote U_T - the set of all admissible controls $u = u(t), t \in T$, with values from the compact $U \subset R^m$.

Consider the ensemble of trajectories of differential inclusion (1), i.e. a multi-valued map $t \rightarrow X(t, D, u(\cdot))$, $t \in T$, where

$$X(t, D, u(\cdot)) = \left\{ \xi \in R^n : \xi = x(t), x(\cdot) \in H(D, u(\cdot)) \right\}, t \in T,$$

$H(D, u(\cdot))$ is the set of absolutely continuous trajectories $x = x(t, x^0, u(\cdot))$ of differential inclusion (1) corresponding to the control $u(\cdot) \in U_T$ and the initial condition $x(t_0) = x^0 \in D$.

For models of the form (1), one of the frequently used criteria for controlling the ensemble of trajectories is the terminal functional $J(x(\cdot)) = g(x(t_1))$, $x(\cdot) \in H(D, u(\cdot))$, where $g(\xi)$ is the given function of the argument $\xi \in R^n$. Usually, in problems of nonsmooth optimization, functions from the class of subdifferentiable or quasidifferentiable functions are selected as the terminal functional [3,12]. The latter includes, in particular, convex and concave functions, as well as functions such as maximum and minimum (for example, $g(\xi) = \max_{\omega \in \Omega} \varphi(\omega, \xi)$, $g(\xi) = \min_{\omega \in \Omega} \varphi(\omega, \xi)$).

Consider the following minimax control problem for the ensemble of trajectories of system (1):

$$\Phi(D, u(\cdot)) = \sup_{\xi \in X(t_1, D, u(\cdot))} g(\xi) \rightarrow \min_{u(\cdot) \in U_T}. \quad (3)$$

Suppose that in problem (2) the function $g(\xi)$, $\xi \in R^n$ has the form

$$g(\xi) = \min_{z \in Z} \max_{m \in M} (z, P\xi + m), \quad (4)$$

where P is a $s \times n$ -matrix, Z and M are compact subsets R^s . This function (4) can be written as a minimum function, i.e. we have

$$g(\xi) = \min_{z \in Z} [(z, P\xi) + \max_{m \in M} (m, z)] = \min_{z \in Z} [(z, P\xi) + C(M, z)], \quad (5)$$

where $C(M, z) = \max_{m \in M} (m, z)$ - support function of the compact set M .

3. The main results

Will be study the necessary and sufficient conditions of optimality for minimax control problem (3). The set $X_T(t, D, u)$ have representation [6]

$$X(t, D, u(\cdot)) = F(t, t_0)D + \int_{t_0}^t F(t, \tau)B(\tau, u(\tau))d\tau, \quad (6)$$

where $F(t, \tau)$ is the fundamental matrix of solutions of the equation

$$\frac{dx}{dt} = A(t)x, t \geq t_0,$$

i.e. $\frac{\partial F(t, \tau)}{\partial t} = A(t)F(t, \tau)$, $F(\tau, \tau) = E$, $\tau \geq t_0$, where E is the identity $n \times n$ matrix.

Assuming that the initial set D is a convex compact, we have the following confirmation.

Theorem 1. The set $X(t, D, u(\cdot)), t \in T$ is a convex compact set from R^n , and its support function is expressed by the formula:

$$C(X(t, D, u(\cdot)), \psi) = C(F(t, t_0)D, \psi) + \int_{t_0}^t C(F(t, \tau)B(\tau, u(\tau)), \psi) d\tau, \psi \in R^n. \quad (7)$$

Given the specifics of defining the terminal function (5), and using the minimax theorem from convex analysis, we have the following representation of the functional in problem (3):

$$\Phi(D, u(\cdot)) = \min_{z \in coZ} [C(X(t_1, D, u(\cdot)), P'z) + C(M, z)], \quad (8)$$

where coZ is the convex hull of the set z .

Using formula (6) and putting it $\psi(t, z) = F'(t_1, t)P'z$, we write equality (7) in the following form:

$$\Phi(D, u(\cdot)) = \min_{z \in coZ} [C(D, \psi(t_0, z)) + C(M, z) + \int_{t_0}^{t_1} C(B(t, u(t)), \psi(t, z)) dt]. \quad (9)$$

Consider the function

$$\mu(z) = C(D, \psi(t_0, z)) + C(M, z) + \int_{t_0}^{t_1} \min_{u \in U} C(B(t, u), \psi(t, z)) dt, z \in coZ. \quad (10)$$

Theorem 2. For optimality control $u^0(t), t \in T$ in problem (3), it is necessary and sufficient that exist $z^0 \in coZ$ such that is the global minimum point of a function $\mu(z)$ of the form (10) and the condition

$$C(B(t, u^0(t)), \psi^0(t)) = \min_{u \in U} C(B(t, u), \psi^0(t)) \quad (11)$$

for almost all $t \in T$, where $\psi^0(t) = \psi(t, z^0)$.

Proof. Necessity. If $u^0(t), t \in T$, is a optimal control in the problem (3), then $\Phi(D, u^0(\cdot)) \leq \Phi(D, u(\cdot)) \quad \forall u(\cdot) \in U_T$, i.e. according to (9),

$$\begin{aligned} \min_{z \in coZ} [C(D, \psi(t_0, z)) + C(M, z) + \int_{t_0}^{t_1} C(B(t, u^0(t)), \psi(t, z)) dt] &\leq \\ &\leq \min_{z \in coZ} [C(D, \psi(t_0, z)) + C(M, z) + \int_{t_0}^{t_1} C(B(t, u(t)), \psi(t, z)) dt], \forall u(\cdot) \in U_T. \end{aligned} \quad (12)$$

We consider the function

$$\eta^0(z) = C(D, \psi(t_0, z)) + C(M, z) + \int_{t_0}^{t_1} C(B(t, u^0(t)), \psi(t, z)) dt.$$

This function is continues, and therefore exist the global minimum point z^0 of a function in convex compact set coZ . According to, from (12) we have

$$\begin{aligned} C(D, \psi(t_0, z^0)) + C(M, z) + \int_{t_0}^{t_1} C(B(t, u^0(t)), \psi(t, z^0)) dt &\leq \\ &\leq C(D, \psi(t_0, z^0)) + C(M, z) + \int_{t_0}^{t_1} C(B(t, u(t)), \psi(t, z^0)) dt, \forall u(\cdot) \in U_T. \end{aligned}$$

Therefore

$$\int_{t_0}^{t_1} C(B(t, u^0(t)), \psi(t, z^0)) dt = \min_{u(\cdot) \in U_T} \int_{t_0}^{t_1} C(B(t, u(t)), \psi(t, z^0)) dt = \int_{t_0}^{t_1} \min_{u \in U} C(B(t, u), \psi(t, z^0)) dt,$$

$$C(B(t, u^0(t)), \psi^0(t)) = \min_{u \in U} C(B(t, u), \psi^0(t))$$

for almost all $t \in T$, $\psi^0(t) = \psi(t, z^0)$, i.e. condition (11) is true.

It easily follows from correlation

$$\begin{aligned} \min_{u(\cdot) \in U_T} \Phi(D, u(\cdot)) &= \eta^0(z^0) = C(D, \psi(t_0, z^0)) + C(M, z^0) + \int_{t_0}^{t_1} C(B(t, u^0(t)), \psi(t, z^0)) dt \geq \\ &\geq C(D, \psi(t_0, z^0)) + C(M, z^0) + \int_{t_0}^{t_1} \min_{u \in U} C(B(t, u), \psi(t, z^0)) dt = \\ &= \mu(z^0) \geq \min_{z \in coZ} \mu(z) = \min_{u(\cdot) \in U_T} \Phi(D, u(\cdot)), \end{aligned}$$

that $z^0 \in coZ$ is the global minimum point of a function $\mu(z)$.

Sufficiently. Let $z^0 \in coZ$ is the global minimum point of the function $\mu(z)$ on coY and satisfy the minimum condition (11) almost everywhere on T . Then:

$$\int_{t_0}^{t_1} C(B(t, u^0(t)), \psi(t, z^0)) dt = \int_{t_0}^{t_1} \min_{u \in U} C(B(t, u), \psi(t, z^0)) dt = \min_{u(\cdot) \in U_T} \int_{t_0}^{t_1} C(B(t, u(t)), \psi(t, z^0)) dt.$$

According to given equation and formula (9) we have

$$\begin{aligned} \Phi(D, u^0(\cdot)) &= \min_{z \in coZ} [C(D, \psi(t_0, z)) + C(M, z) + \int_{t_0}^{t_1} C(B(t, u^0(t)), \psi(t, z)) dt] \leq \\ &\leq C(D, \psi(t_0, z^0)) + C(M, z^0) + \int_{t_0}^{t_1} C(B(t, u^0(t)), \psi(t, z^0)) dt = \min_{z \in coZ} \mu(z). \end{aligned}$$

As $\min_{z \in coZ} \mu(z) = \min_{u(\cdot) \in U_T} \Phi(D, u(\cdot))$, then from given relation it follows that

$$\Phi(D, u^0(\cdot)) = \min_{u(\cdot) \in U_T} \Phi(D, u(\cdot)),$$

i.e. $u^0(t), t \in T$ is a optimal control in problem (2). The theorem is proved.

4. Discussion of the results and conclusion

In the work, the problem of controlling an ensemble of trajectories of differential inclusion in the form of a nonsmooth minimax type problem is studied.

When studying the minimax problem (3), a representation of the form (6) of the ensemble of trajectories was used to obtain formula (7) for the support function of a convex compact set $X(t_1, D, u)$. Further, as a result of applying formula (7) and the minimax theorem from convex analysis, we obtained formula (9) for the optimized functional (3). A new representation (9) of the quality control criterion for the ensemble of trajectories made it possible to reduce the minimax problem (3) to the repeated minimization problem of the function $\mu(z)$ of the form (10).

According to the given optimality conditions, in order to construct an optimal control, one must first solve the finite-dimensional optimization problem:

$$\mu(z) \rightarrow \min, z \in coZ.$$

If $z^0 \in coZ$ is a solution to this problem, then the optimal control $u^0(t), t \in T$ is determined from condition (10), i.e. as a result of resolving a parameterized optimization problem:

$$C(B(t, u), \psi^0(t)) \rightarrow \min, u \in U, t \in T.$$

And this shows that the results obtained constitute the theoretical basis for the algorithm for constructing optimal control in the considered minimax optimal control problem.

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