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Rational Roots of a Polynomial With Integer Coefficients. Performing Actions on them

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ABSTRACT

This article describes in detail the rational roots of polynomials with all the coefficients and the operations on them.

KEYWORDS: *theorem, proof, mathematical property, polynomial, rational roots, coefficient, etc.*

At the time when the interest in exact sciences is increasing worldwide, the teaching of these subjects is improving in our country, and our president Shavkat Mirziyoyev said in his lectures that "Mathematics is the basis of all exact sciences. A child who knows this subject well will grow up to be intelligent, broad-minded, and will work successfully in any field. Today, when science and technology are developing rapidly, the volume of scientific knowledge, understanding and imagination is increasing sharply. For this reason, it is an urgent issue that we teach mathematics to the future youth of the Republic of Uzbekistan. The topic of complex numbers is familiar to us from the school mathematics course. This subject is also taught as a separate subject in higher education institutions. This field is widely used in many fields of technology and production. It is known that mathematics, as a science, occupies a high place in forming the thinking of students, finding solutions to various problematic situations, and understanding digitization technologies. Today's modern education system itself is based on the combined influence of ideas and methods of various sciences. Naturally, this is a relationship related to mathematical thinking in many ways. The inclusion of mathematics in the school curriculum serves to deepen the learning process and prepare students for life and work in the conditions of market relations. It is known that practical issues of economics are solved by mathematicians in most cases. Addition, subtraction, multiplication, and exponentiation of complex numbers are performed in the same way as for polynomials. The operations of division and extraction of the root are defined as the opposite operations of the operations of multiplication and rising to the level, respectively.

The subject matter of this work is quadratic and cubic polynomials with integral coefficients; which also has all of the roots being integers. The purpose of this work is to determine precise (i.e. necessary and sufficient) coefficient conditions in order that a quadratic or a cubic polynomial has integer roots only. The results of this paper are expressed in Theorems 3, 4, and 5. The level of the material in this article is such that a good second or third year mathematics major; with some exposure to number theory (especially the early part of an introductory course in elementary number theory); can comfortably come to terms with. Let us outline the organization of this article.

It follows immediately from Th. 1(b) (with a straightforward expansion/calculation), that if $t(x) = a_2x^2 + a_1x + a_0$ is a quadratic trinomial with complex number coefficients a_2, a_1, a_0; and roots r_1 and r_2. Then $t(x) = a_2(x - r_1)(x - r_2)$; and consequently

$$r_1 + r_2 = -\frac{a_1}{a_2} \text{ and } r_1 r_2 = \frac{a_0}{a_2} \qquad (1)$$

Likewise, if $c(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ is a cubic polynomial function with complex number coefficients a_3, a_2, a_1, a_0; and roots r_1, r_2, r_3. Then Theorem 1, part (b) implies that

$c(x) = a_3(x - r_1)(x - r_2)(x - r_3)$; and accordingly,

$$\begin{Bmatrix} r_1 + r_2 + r_3 = -\frac{a_2}{a_3} \\ r_1r_2 + r_2r_3 + r_3r_1 = \frac{a_1}{a_3} \\ r_1r_2r_3 = -\frac{a_1}{a_3} \end{Bmatrix} \qquad (2)$$

There are three lemmas and five theorems in total. The results expressed in Theorems 4 and 5; are not found in standard undergraduate texts (in the United States) covering material of the first two years of the undergraduate mathematics curriculum. But some of these results might be found in more obscure analogous books (probably out of print) around the globe. The three lemmas are number theory lemmas and can be found in Section 3. Lemma 1 is known as Euclid's lemma; it is an extremely well known lemma and can be found in pretty much every introductory book in elementary number theory. We use Lemma 1 to establish Lemma 3; which is in turn used in the first proof of Theorem 3 (we offer two proofs for Th. 3). Lemma 2, also known as the nth power lemma, is also very well known; but perhaps a bit less than Lemma 1. A reference for Lemma 1 can be found in. A reference for Lemma 2 can be found in (it is also listed as an exercise in). Lemma 2 is used in the second proof of Theorem 3 and also toward establishing Theorem 4. Theorem 3, in Section 4; states necessary and sufficient conditions for a quadratic polynomial with integer coefficients to have two integral roots or zeros. The result in Th. 3 is generally recognizable, but the interested reader may not find it in standard undergraduate texts. We offer two proofs to Theorem 3. Theorem 4, in Section 6; is the one that (most likely) the reader will be the least familiar with. Theorem 4 gives precise (i.e. necessary and sufficient) coefficient conditions, in order that a monic (i.e. with leading coefficient 1) cubic polynomial with integer coefficients; have one double (i.e. multiplicity 2) integral root, as well as another integer root.

The problem of finding two polynomials P(x) and Q(x) of a given degree n in a single variable x that have all rational roots and differ by a non-zero constant is investigated. It is shown that the problem reduces to considering only polynomials with integer roots. The cases n < 4 are solved generically. For n = 4 the case of polynomials whose roots come in pairs (a,-a) is solved. For n = 5 an infinite number of inequivalent solutions are found with the ansatz P(x) = -Q(-x). For n = 6 an infinite number of solutions are also found. Finally for n = 8 we find solitary examples. This also solves the problem of finding two polynomials of degree n that fully factorise into linear factors with integer coefficients such that the difference is one. This problem is also equivalent to finding two polynomials of degree n that can be fully factorised into linear factors with integer coefficients and that differ by one. If we have two such polynomials then they have rational roots. Conversely, if we have two polynomials P(x) and Q(x) with integer roots that differ by a constant P(x) – Q(x) = c, then substitute x' = x – s1 where s1 is a root of Q(x) so that Q'(x') = Q(x' + s1) has a factor of x' and therefore the product of the roots of P'(x') = P(x' + s1) is c. Now make a second substitution to define P''(x'') = (1/c)P'(cx'') and Q''(x'') = (1/c)Q'(cx''). It can be verified that P''(x'') and Q''(x'') factorise into linear factors with integer coefficients and that P''(x'') - Q''(x'') = 1.

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