

## Irrational Numbers. the Existence of a Rational Number Whose Square is Two

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### ABSTRACT

*This article describes in detail examples and problems that prove the existence of irrational numbers and a rational number whose square is equal to two.*

**KEYWORDS:** *irrational numbers, periodic division, rational numbers, square, theorem, proof, etc.*

In our study of decimals, we have looked separately at non-periodic decimals. Such fractions appear in decimal measure of the length of segments that are not comparable to a single segment. We also noted that non-recurring decimals cannot be converted to simple fractions (see Converting simple fractions to decimals and vice versa), so these numbers are not rational numbers, they are so-called irrational numbers represents The sound definition allows to bring examples of irrational numbers. For example, the infinite non-periodic decimal  $4.10110011100011110000\dots$  (the number of ones and zeros increases by one each time) is a rational number. Let's give another example of an irrational number:  $-22.353335333335\dots$  (the number of triplets dividing the eight is increased by two every time). It should be noted that irrational numbers are extremely rare in the form of non-periodic decimals. Usually they are found in the form etc., as well as in the form of specially inserted letters. the most famous examples of irrational numbers in such notation are the arithmetic square root of two, the number "pi"  $p=3.141592\dots$ ,  $e=2.718281\dots$  and the golden number.

Also, any composition of rational numbers connected by arithmetic operations (+, -, ·, :) is not an irrational number. This is because the sum, difference, product, and quotient of two rational numbers is a rational number. Examples are values of expressions and rational numbers. We note here that if such expressions contain one irrational number among the rational numbers, then the value of the entire expression is an irrational number. For example, the number in the expression is irrational and the rest of the numbers are rational, so it is an irrational number. If it is a rational number, then the rationality of the number follows, but it is not rational. If the expression for which the number is given contains several irrational numbers, root symbols, logarithms, trigonometric functions,  $p$ ,  $e$ , etc., then in each specific case it is required to prove the irrationality or rationality of the given number. However, there are a number of already obtained results that can be used. Let's list the main ones. It is proved that the  $k$ th root of an integer is a rational number, if the number under the root is the  $k$ th power of another integer; otherwise such a root defines an irrational number. For example, the numbers  $\sqrt{7}$  and  $\sqrt[5]{15}$  are irrational because there is no integer whose square is 7, and there is no integer whose power to the fifth power is 15. The numbers  $\log_2 3$  and  $\log_3 2$  are not irrational because and. As for logarithms, sometimes their irrationality can be proved by contradiction. For example, let's prove that  $\log_2 3$  is an irrational number. Let's say that  $\log_2 3$  is not an irrational number, but a rational number, that is, it can be expressed as a simple fraction  $m/n$ . and let write the following chain of equality:  $2^{m/n} = 3$ . The last equality is impossible, because it has an odd number on the left, and an even number on the right. So we come to a contradiction, which proves that our assumption is wrong and that  $\log_2 3$  is an irrational number.

It is also proved that the number  $e^a$  is irrational for any nonzero rational  $a$ , and the number  $p^z$  is irrational for any nonzero integer  $z$ . For example, numbers  $\sqrt{2}$  and  $\sqrt{3}$  are irrational. For any rational and nonzero value of the argument, the trigonometric functions  $\sin$ ,  $\cos$ ,  $\tan$ , and  $\cot$  are also irrational

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numbers. For example,  $\sin 1$ ,  $\operatorname{tg}(-4)$ ,  $\cos 5,7$ , are irrational numbers. There are other proven results, but we will limit ourselves to those already listed. It should also be said that algebraic numbers and transcendental numbers are related to the theory in the proof of the above results. In conclusion, we emphasize that one should not make hasty conclusions about the irrationality of the given numbers. For example, it is obvious that an irrational number is an irrational number. However, this is not always the case. As a confirmation of the received fact, we present the degree. It is known that it is an irrational number, and it has also been proved that it is an irrational number, but it is also a rational number. You can also give examples of irrational numbers whose sum, subtraction, product, and quotient are all rational numbers. In addition, the rationality or irrationality of the numbers  $p+e$ ,  $p-e$ ,  $p e$ ,  $p p$ ,  $p e$  and many others has not yet been proven. Irrational numbers are infinitely aperiodic. The need to introduce this concept is related to the inadequacy of the previously existing concepts of real or real, whole, natural and rational numbers to solve newly emerging problems. For example, to calculate what the square of 2 is, you need to use non-repeating decimals. In addition, many of the simplest equations have no solution without introducing the concept of an irrational number. For the first time, in one way or another, Indian mathematicians encountered this phenomenon in the 7th century, when it was discovered. square roots of some quantities cannot be specified exactly. And the first proof of the existence of such numbers belongs to Hypas of Pythagoras, who did it during the study of an equilateral triangle. Other scientists who lived before our era made a significant contribution to the study of this collection. The introduction of the concept of irrational numbers led to a revision of the existing mathematical system, so they are very important. There are still many unresolved issues with this collection. There are criteria such as measure of irrationality and normality of the number. Mathematicians continue to examine the most important examples for their belonging to one or another group. For example,  $e$  is a normal number, that is, it is considered that the probability of different numbers appearing in its input is the same. As for  $\pi$ , research related to it is still ongoing. A measure of irrationality is a value that shows how closely a given number can be approximated by rational numbers.

2	3	5	7	11	13	17	19	23	29	31	37
41	43	47	53	59	61	67	71	73	79	83	89
97	101	103	107	109	113	127	131	137	139	149	151
157	163	167	173	179	181	191	193	197	199	211	223
227	229	233	239	241	251	257	263	269	271	277	281
283	293	307	311	313	317	331	337	347	349	353	359
367	373	379	383	389	397	401	409	419	421	431	433
439	443	449	457	461	463	467	479	487	491	499	503
509	521	523	541	547	557	563	569	571	577	587	593
599	601	607	613	617	619	631	641	643	647	653	659
661	673	677	683	691	701	709	719	727	733	739	743
751	757	761	769	773	787	797	809	811	821	823	827
829	839	853	857	859	863	877	881	883	887	907	911
919	929	937	941	947	953	967	971	977	983	991	997

If we pay attention, in the process of solving problems related to prime numbers, the most beautiful field of mathematics - "Number theory" was created as a result of the passionate work of great mathematicians. It is interesting to note that no matter how complicated the problems of number theory are, the content of the problem is understandable even for school or high school students. For example, the Goldbach problem. In 1742, in a letter to L. Euler, a member of the St. Petersburg Academy, Goldbach stated the following hypothesis: Any natural number greater than five can be expressed as the sum of at most 3 prime numbers. Euler did not solve the Goldbach problem, but he showed that it would suffice to prove that any even number not less than 4 is the sum of two prime numbers. The concept of irrational numbers was implicitly accepted by Indian mathematicians in the

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7th century BCE, when Manava (c. 750 BCE - 690 BCE) discovered that the square roots of certain natural numbers, such as 2 and 61, could not be expressed exactly. The first evidence for the existence of irrational numbers is generally attributed to the Pythagorean Hypas of Metapontus (c. 500 BC). At the time of the Pythagoreans, it was believed that there was a single unit of length sufficiently small and indivisible, which was the whole number included in any segment. It is not clear which number Hippasus proved to be irrational. According to legend, he discovered the sides of the pentagram while studying their lengths. Therefore, it is reasonable to assume that it was the golden ratio. Greek mathematicians called this ratio of unequal quantities alogos (indescribable), but according to legend, Hippasus was not given due respect. According to legend, Hippasus made the discovery while on a sea voyage and was thrown by other Pythagoreans "to create an element of the universe that refutes the doctrine that all beings in the universe are reducible to whole numbers and their ratios." "The discovery of Hippasus created a serious problem for Pythagorean mathematics, and destroyed the assumption on which the whole theory was based, that numbers and geometric bodies are inseparable from each other.

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