Propagation of Plane and Spherical Waves in a Medium with Broken Unloading

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ABSTRACT

This article provides information about the laws of distribution of waves, taking into account a sphere with a broken unloading, as well as their solutions.

KEYWORDS: *state diagram, unloading, velocity, boundary conditions, pressure isobar, wave equation.*

If the state diagram of the medium during unloading has a broken line (Fig. 3) consisting of two straight lines, then the results of the previous paragraph are valid as long as $P(r,t) \geq P^{**}$. Therefore, based on the physical plane (r,t), the surface $r=R_1^*(t)$ is first determined, in which $P=P^{**}$, and then the distribution of velocity and strain on it is found from the calculations. Calculations show that the pressure at the shock wave front decays more weakly than at the cavity. In this regard, the pressure isobar turns out to be elongated towards the spatial coordinate r (Fig. 1).

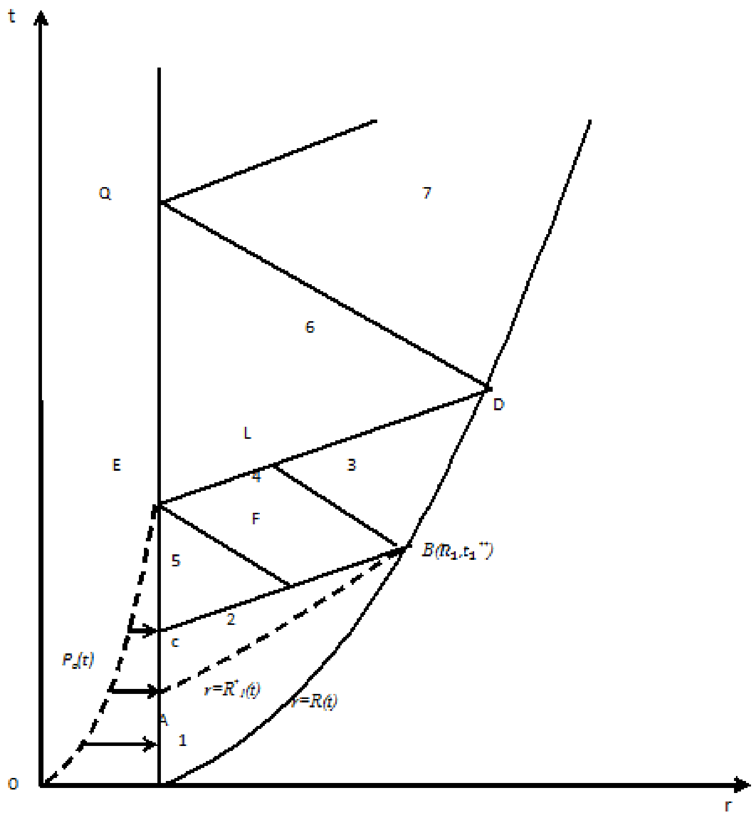

Figure 1. Pressure isobar during unloading.

Depending on the value of the rate of "Unloading strain" $C_{p1} = \sqrt{E_1/\mathcal{P}_0}$ (Fig.3) ($E_1 < E$) there may be cases $a,\ \delta$. If $\dot{R}_1(t) < C_{p1}$, then case *a* (Fig. 1) is realized, and for $\dot{R}_1(t) > C_{p1}$, case δ is realized (Fig. 2).

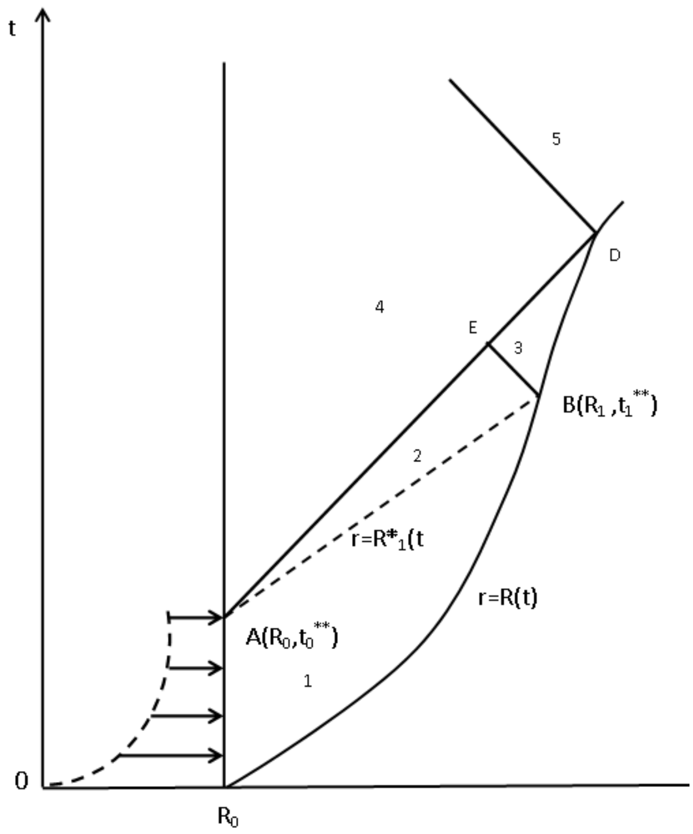

Figure 2. Unloading cases

We assume the solution of the problem for the case a . In this case, region 2 is bounded by a non-characteristic surface *AB*, where $P\ (r,t) = P^{**} = const$, a characteristic of the positive direction *BC* and a layer boundary AC (Fig. 4). Note that the solution to this problem in region I, where $P = (r,t) \geq P^{**}$ is constructed, will be used to obtain the corresponding solution to the problem in the subsequent region 2. [1].

In region 2, this problem has boundary conditions

$$\left.\begin{array}{c} P(r,t) = P^{**} = const, \\ U(r,t) = U^{**}(t), \\ \frac{\partial U}{\partial r} + \nu \frac{U}{r} = -\frac{\partial \varepsilon^{*}(t)}{\partial t} \end{array}\right\} \text{при } r = R_1(t) \qquad (1)$$

And the equation of state of the medium

$$P(r,t) = P^{**} + E_1(\mathcal{E} - \mathcal{E}^{**}), \qquad (2)$$

where, $E_1 = p_0 C_{p1}^2$, P^{**} −value given by the diagram $P\infty\mathcal{E}$. In the plane case (ν=0), the wave equation for region 2 is written as:

$$\frac{\partial^2 U}{\partial t^2} - C_{p1}^2 \frac{\partial^2 U}{\partial r^2} = 0, \qquad (3)$$

Which has a solution

$$U(r,t) = F_1\big(r - C_{p1}t\big) + F_2(r + C_{p1}t) \qquad (4)$$

Substituting (4) into the last two conditions (1) and performing calculations similar to those for the problem with linear unloading, we obtain:

$$U(r,t) = U^{**}(R_0, t_0^{**}) - \frac{1}{2C_{p1}}\left\{\int_{Z_{30}}^{r-C_{p1}t}\left[\left(\dot{R}_1\left(F(Z_1)\right) + C_{p1}\right)\cdot \dot{E}^{**}\left(F(Z_1)\right) + \dot{U}^{**}\left(F(Z_1)\right)\right]dZ_1 + \int_{Z_{40}}^{r+C_{p1}t}\left[\left(\dot{R}_1\left(F(Z_2)\right) - C_{p1}\right)\cdot \dot{E}^{**}\left(F(Z_2)\right) + \dot{U}^{**}\left(F(Z_2)\right)\right]dZ_2\right\}. \quad (5)$$

where, $Z_{30,40} = R_0 \mp C_{p1}t_0$, $F(Z_i)$ $(i = 1,2)$ is the root of the equation $R_1^*(t) \mp C_{p1}t = Z_i$ _i with respect to time t.

After integrating the first equation, taking into account (5), the load P_0 (t) at the layer boundary AC is expressed by the formula:

$$P_0(t) = P^{**} - \frac{\mathcal{P}_0}{2}\int_{R_1(t)}^{R_0}\left\{\left[\dot{R}_1\left(F\left(r - C_{p1}t\right)\right) + C_{p1}\right)\cdot \dot{\mathcal{E}}^{**}\left(F\left(r - C_{p1}t\right)\right) + \dot{U}^{**}\left(F\left(r - C_{p1}t\right)\right)\right] + \left[\left(\dot{R}_1\left(F\left(r + C_{p1}t\right)\right) - C_{p1}\right)\cdot \dot{\mathcal{E}}^{**}\left(F\left(r + C_{p1}t\right)\right) + \dot{U}^{**}\left(F\left(r + C_{p1}t\right)\right)\right]\right\}dr. \quad (5)$$

For a spherical wave (*V=2*) in region 2, taking into account (1), the solution of equation (3) with the coefficient C_p replaced by C_{p1} is represented as:

$$U(r,t) = \frac{1}{r}\left\{-\int_{Z_{10}}^{r-C_{p1}t} d\xi_2 \int_{Z_{10}}^{\xi_2}\phi(\xi_1)\,d\xi_1 - \int_{Z_{20}}^{r+C_{p1}t} R_1[F(Z_2)]\cdot \dot{\mathcal{E}}^{**}\left(F(Z_2)\right)dZ_2 + \int_{Z_{20}}^{r+C_{p1}t} dZ_2 \int_{Z_{10}}^{R_1[F(Z_2)]-C_{p1}F(Z_2)} \phi(\xi_1)\,d\xi_1\right\}$$
$$- \frac{1}{r^2}\left\{-\int_{Z_{10}}^{r-C_{p1}t} d\,Z_1 \int_{Z_{10}}^{Z_1} d\xi_2 \int_{Z_{10}}^{\xi_2}\phi(\xi_1)d\xi_1 - \int_{Z_{20}}^{r+C_{p1}t} d\xi_2 \int_{Z_{20}}^{\xi_2} R_1[F(\xi_2)]\cdot \dot{\mathcal{E}}^{**}[F(\xi_2)]d\xi_2 + \int_{Z_{20}}^{r+C_{p1}t} d\xi_3 \int_{Z_{20}}^{\xi_3} d\xi_2 \int_{Z_{20}}^{R_1[F(\xi_2)]-C_{p1}F(\xi_2)} \phi(\xi_1)d\xi_1\right\} + \frac{m_3 C_{p1}t}{r^2} - \frac{n_3}{r^2}.$$

$$\Phi(\xi_1) = -\frac{\dot{R}_1^3[F(Z_1)]}{2C_{p1}\cdot R_1[F(Z_1)](\dot{R}_1 - C_{p1})}\left\{\frac{\ddot{U}[F(Z_1)]\cdot R_1^2}{\dot{R}_1^3} + \frac{4R_1\dot{U}^{**}[F(Z_1)]}{\dot{R}_1^2} + 2\left(\frac{1}{\dot{R}_1} + \frac{R_1\cdot \ddot{R}_1}{\dot{R}_1^3}\right)U^{**}[F(Z_1)] + \frac{R_1}{\dot{R}_1}\left[\left(2 - \frac{C_{p1}}{\dot{R}_1}\right)\left(1 + \frac{C_{p1}}{\dot{R}_1}\right) + \frac{R_1\cdot \ddot{R}_1}{\dot{R}_1^2}\right]\dot{\mathcal{E}}^{**}\left(F(Z_1)\right) + \frac{R_1^2}{\dot{R}_1^2}\left(1 + \frac{C_{p1}}{\dot{R}_1}\right)\cdot \dot{\mathcal{E}}^{**}\left(F(Z_1)\right)\right\}. \quad (6)$$

where: $Z_{10,20}=R_1(t_0) \mp C_{p1}t_0$, $m_3 = (C_2 - C_3)$, $n_3 = (C_4 - C_5)$ are arbitrary integration constants determined from the condition $U(r,t) = U_0(t_0)$, $\dot{U}(r,t) = \dot{U}_0(t_0)$, are expressed as dependencies:

$$m_3 = R_0^2 \left[\frac{\dot{U}_0(t_0)}{C_{p1}} + \dot{\mathcal{E}}^{**}(t_0)\right],$$

$$n_3 = C_{p1}t_0R_0^2 \left[\frac{\dot{U}_0(t_0)}{C_{p1}} + \dot{\mathcal{E}}^{**}(t_0)\right] - R_0^2U_0(t_0), \qquad (7)$$

The formula for the load, taking into account (1) and (4), has the form:

$$P_0(t) = P^{**} + p_0C_{p1}\left\{\int_{R_0}^{R_1(t)}\left\{\int_{z_{10}}^{r-C_{p1}t}\phi(\xi_1)d\xi_1 - R_1[F(r+C_{p1}t)]\dot{\mathcal{E}}^{**}[F(r+C_{p1}t)] + \int_{z_{10}}^{R_1[F(r+C_{p1}t)]-c_{p1}F(r+C_{p1}t)}\phi(\xi_1)d\xi_1\right\}dr - \int_{R_0}^{R_1(t)}\frac{1}{r^2}\left\{\int_{z_{10}}^{r-C_{p1}t}d\xi_2\int_{z_{10}}^{\xi_2}\phi(\xi_1)d\xi_1 - \int_{z_{20}}^{r+C_{p1}t}R_1[F(\xi_2)]\cdot\dot{\mathcal{E}}^{**}[F(\xi_2)]d\xi_2 + \int_{z_{20}}^{r+C_{p1}t}d\xi_2\int_{z_{20}}^{R_1[F(\xi_2)]-C_{p1}F(\xi_2)}\phi(\xi_1)d\xi_1 - m_3\right\}dr\right\}.$$
(8)

Thus, the solutions of the plane and spherical problems in region 2, taking into account (3), (4) and (5), (6), are completely obtained.

Note that the solution of the problem for both a plane and a spherical wave in region 3 is constructed in the same way as in region I, with the only difference that in region 3 Young's modulus $\boldsymbol{E_1}$ takes place [2]. In areas 4 and 5, which are limited by the characteristics of the positive and negative directions, as well as the boundary of the layer *CE* (Fig. 1), the Goursat problem is obtained, the construction of a solution to which is not difficult [3].

Based on the solution of the problem in area 5, the profile of the load on the *CE* is determined. For subsequent areas 6, 7, etc., the problem is solved in a similar way until $\boldsymbol{P_0(t)} \geq 0$ [4].

In the case δ (Fig. 2), the solutions of the problems of plane and spherical waves in region 2 do not differ from the case α and they are mathematically identical. However, in the case of α, at the boundary of the layer, we have a section of application of the load *AC*, while in the case of δ it is absent. In this regard, in the case of α, it is required to determine the load profile on *AC*, and in the case of δ, an additional region 3 arises, where it is necessary to find the forms of the shock wave front in section *BD*.

In conclusion, we can say that the solution of problems for a plane and spherical wave in region 3 (Fig. 2), taking into account the corresponding boundary conditions on the characteristic *BE*, mathematically reduces to a boundary problem, where there was a linear unloading of the medium. Therefore, apparently there is no need to present here the solution of the above problem.

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Conclusions:

In conclusion, we can say that the solution of problems for a plane and spherical wave in region 3 (Fig. 2), taking into account the corresponding boundary conditions on the characteristic *BE*, mathematically reduces to a boundary problem, where there was a linear unloading of the medium. Therefore, apparently there is no need to present here the solution of the above problem.

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