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Equations of Transversal Vibration of a Two-Layer Viscoelastic Plate of Constant Thickness

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ABSTRACT

This article discusses the analysis of the general equations of the transverse oscillation of a piecewise-homogeneous viscoelastic plate obtained in the "Oscillations of two-layer plates of constant thickness" article [1].

In this paper, on the basis of a mathematical method, an approximate theory of oscillation of piecewise homogeneous plates is developed, based on considering the plate as a three-dimensional body, on the exact formulation of the three-dimensional mathematical boundary value problem of oscillation under external forces causing transverse oscillations.The theoretical results obtained for solving dynamic problems of transverse vibrations of piecewise homogeneous two-layer plates of constant thickness, taking into account the viscous properties of their material, make it possible to more accurately calculate the stress-strain state of the plates under non-stationary external loads.In the present work on the basis of a mathematical method, the approached theory of fluctuation of the two-layer plates, based on plate consideration as three dimensional body, on exact statement of a three dimensional mathematical regional problem of fluctuation is stood at the external efforts causing cross-section fluctuations.The received theoretical results for the decision of dynamic problems of cross-section fluctuation of piecewise homogeneous two-layer plates of a constant thickness taking into account viscous properties of their material allow to count more precisely the is intense-deformed status of plates at non-stationary external loadings.

KEYWORDS: *cross-section fluctuations, a two-layer plate, the integro-differential equation, the fluctuation equations.*

The general equations of oscillations of piecewise homogeneous viscoelastic plates of constant thickness, described in [1], are complex in structure and contain derivatives of any order in coordinates x, y, and time t, and therefore are not suitable for solving applied problems and performing engineering calculations.

To solve applied problems, instead of general equations, it is advisable to use approximate ones, which include one or another finite order in derivatives.

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The classical equations for the transverse oscillation of a plate contain derivatives of no higher than the 4th order, and for piecewise homogeneous or two-layer plates, the simplest approximate oscillation equation is a sixth order equation.

If we confine ourselves to the first two terms in operators (3.8) given in [1], then from equation (3.11) we obtain an approximate integro-differential equation.

$$Q_1\left(\frac{\partial^4 W}{\partial t^4}\right)+Q_2\left(\Delta\frac{\partial^2 W}{\partial t^2}\right)+Q_3\left(\Delta^2 W\right)+Q_4\left(\frac{\partial^6 W}{\partial t^6}\right)+Q_5\left(\Delta\frac{\partial^4 W}{\partial t^4}\right)+ \\ +Q_6\left(\Delta^2\frac{\partial^2 W}{\partial t^2}\right)+Q_7\left(\Delta^3 W\right)=F_1(x,y,t). \quad (1)$$

where the operators Q_j and $F_1(x,y,t)$ are equal:

$$Q_1=M_1^{-2}(h_0\rho_0+h_1\rho_1)^2;$$
$$Q_2=-2M_1^{-2}(2(h_0P_2D_0+h_1D_1)(h_0\rho_0+h_1\rho_1)+ \\ +(P_2-1)(h_0\rho_0(h_0+h_1)-(h_0^2D_0\rho_0+h_1^2D_1\rho_1)));$$
$$Q_3=4(P_2-1)(h_0^2P_2D_0+h_1^2D_1+h_1^2D_1+2h_0h_1P_2D_0);\,(2)$$
$$Q_4=-\frac{1}{6}M_1^{-2}(h_0^2\rho_0M_0^{-1}(3h_1^2\rho_1^2+h_0\rho_0(h_0\rho_0+4h_1\rho_1))(2-D_0)+ \\ +h_1^2\rho_1M_1^{-1}(3h_0^2\rho_0^2+h_1\rho_1(h_1\rho_1+4h_0\rho_0))(2-D_1));$$
$$Q_5=-\frac{1}{6}M_1^{-2}(h_0^2P_2\rho_0^2M_0^{-2}(2P_2(4D_0(1-D_0)+(P_2-1)(4+D_0^2))- \\ -h_1^4\rho_1^2M_1^{-2}(2(4D_1^2-4D_1-1)-(P_2-1)D_1(2-D_1))+ \\ +6h_0^2h_1^2(\rho_0\rho_1M_0^{-1}M_1^{-1}(4(P_2^2D_0+D_1)+(P_2-1)(2P_2(1-D_0)-P_2D_1(2-D_0) \\ +D_1(1+D_0)))++M_1^{-1}(\rho_0^2+\rho_1^2))+ \quad (2) \\ +2P_2h_0h_1(2\rho_0\rho_1M_0^{-1}M_1^{-1})(h_0^2(2+4D_0-D_0^2)+h_1^2(2P_2-P_2D_1+5D_1-D_1^2))+ \\ +h_0^2h_1^2M_0^{-2}((P_2-1)(4-3D_0)+2D_1(4-D_0))+2h_1^2\rho_1^2M_1^{-2}D_0(4-D_1));$$

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$$Q_6=\frac{1}{3}M_1^{-2}(h_0^{\ 2}P_2\rho_0M_0^{-1}(2P_2((P_2-1)(2+9D_0-3D_0^2))-2D_0(1-3P_2+4D_0))+$$
$$+h_1^4\rho_1M_1^{-1}(4D_1\,(1-2D_1)-4D_1+(P_2-1)D_1(3-D_1))+$$
$$+3h_0^2h_1^2((4P_2D_0(P_2(1-D_1)-D_1)-(P_2-1)(2(P_2-1)D_1(1-D_0)-$$
$$+P_2(2-D_0-2D_0D_1)))\rho_0M_0^{-1}+(4D_1(1+D_0+P_2D_0)-(P_2-1)(6D_0D_1(P_2-1)-$$
$$-6P_2D_0+D_1))\rho_1M_1^{-1})-2h_0h_1P_2(\rho_0M_0^{-1}(2h_0^{\ 2}((P_2-1)(D_0^2-2D_0-1)-$$
$$-2D_1(1+D_0))-h_1^2(2(P_2-1)+D_1(P_2+3))))-$$
$$-4\rho_1M_1^{-1}(h_0^{\ 2}+h_1^{\ 2})(2(P_2-1)(1-D_1)+P_2D_1+(1+D_1))))));$$

$$Q_7=\frac{2}{3}(h_0^4P_2D_0(4D_0-5(P_2-1)+h_1^4D_1\,(4D_1-(P_2-1))-$$
$$+3h_0^2h_1^2(8P_2D_0D_1-(P_2-1)((2(P_2+1)D_0D_1-3P_2D_0-D_1(1-D_1)))-$$
$$-4h_0h_1P_2D_0(h_0^{\ 2}(P_2-1)+2D_1)+h_1^2(2(P_2-1)+(P_2+1)D_0)));$$

$$F_1(x,y,t)=M_1^{-2}\frac{\partial^2}{\partial t^2}((h_0\rho_0+h_1\rho_1)(f_z^{(0)}-f_z^{(1)}))+$$
$$+(h_0+h_1)(h_1\rho_1(\frac{\partial^2 f_{xz}^{(0)}}{\partial x^2}+\frac{\partial^2 f_{yz}^{(0)}}{\partial y^2})+h_0\rho_0(\frac{\partial^2 f_{xz}^{(1)}}{\partial x^2}+\frac{\partial^2 f_{yz}^{(1)}}{\partial y^2}))+$$
$$+(h_0^2D_0\rho_0+h_1^2D_1\rho_1)((\frac{\partial^2 f_{xz}^{(0)}}{\partial x^2}+\frac{\partial^2 f_{yz}^{(0)}}{\partial y^2})-(\frac{\partial^2 f_{xz}^{(1)}}{\partial x^2}+\frac{\partial^2 f_{yz}^{(1)}}{\partial y^2}))- \qquad (3)$$
$$-2\Delta(2M_1^{-2}((h_0P_2D_0+h_1D_1)(M_0f_z^{(0)}-M_1f_z^{(1)}))+$$
$$+2P_2h_0h_1)(D_0M_0^{-1}(\frac{\partial^2 f_{xz}^{(0)}}{\partial x^2}+\frac{\partial^2 f_{yz}^{(0)}}{\partial y^2})+D_1M_1^{-1}(\frac{\partial^2 f_{xz}^{(1)}}{\partial x^2}+\frac{\partial^2 f_{yz}^{(1)}}{\partial y^2}))+$$
$$+M_1^{-1}(h_0^2P_2D_0+h_1^2D_1)(\frac{\partial^2 f_{xz}^{(0)}}{\partial x^2}+\frac{\partial^2 f_{yz}^{(0)}}{\partial y^2})+(\frac{\partial^2 f_{xz}^{(1)}}{\partial x^2}+\frac{\partial^2 f_{yz}^{(1)}}{\partial y^2})).$$

If the plate is homogeneous, and W is the transverse displacement of the points of the “middle” surface - the plane of the plate, then in this case the dependences

$$N_0=N_1;\ \ M_0=M_1;\ \ P_2=1;\ \ h_0=h_1;\ \ C_0=C_1;\ \ D_0=D_1.$$

and equation (1) becomes the equation

$$((1-C_0)^2\lambda_{10}^{(1)}+(1+C_0)^2\Delta)((\lambda_{20}^{(1)}+\Delta)+$$
$$+\frac{h_0^2}{6}((3D_0(\lambda_{20}^{(1)}+\Delta)^2)+4D_0\lambda_{20}^{(1)}\Delta)+4\lambda_{10}^{(1)}(\lambda_{20}^{(1)}+\Delta)))(W)= \qquad (4)$$
$$=\frac{1}{h_0}(M_0^{-2}\frac{\partial^2}{\partial t^2}((f_z)+h_0(\frac{\partial^2 f_{xz}^{(0)}}{\partial x^2}+\frac{\partial^2 f_{yz}^{(0)}}{\partial y^2}))-$$

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$$-4D_0M_0^{-1}\Delta((f_z)+h_0\frac{\partial^2 f_{xz}^{(1)}}{\partial x^2}+\frac{\partial^2 f_{yz}^{(1)}}{\partial y^2})))\ (4)$$

where on the left is the product of two operators: the first describes the process of longitudinal oscillation, and the second describes the transverse oscillation.

Similarly, an approximate equation is introduced from the general equation (1.3.12) given in [1] and we obtain for $\left(\frac{\partial U_1}{\partial y}-\frac{\partial V_1}{\partial x}\right)$

$$\begin{aligned}&(G_1\frac{\partial}{\partial t^2}+G_2\Delta+G_3\frac{\partial^4}{\partial t^4}+G_4\Delta\frac{\partial^2}{\partial t^2}+G_5\Delta^2+G_6\frac{\partial^6}{\partial t^6}+\\&+G_7\Delta+G_8\Delta^2+G_9\Delta^3)(\frac{\partial U_1}{\partial y}-\frac{\partial V_1}{\partial x})=F_2(x,y,t),\end{aligned}\quad (5)$$

where the operators G_j and $F_2(x,y,t)$ are equal:

$$G_1=M_1^{-1}(h_0\rho_0+h_1\rho_1)\ ;$$

$$G_2=-(h_0P_2+h_1)\ ;$$

$$G_3=\frac{1}{6}M_1^{-2}(h_0^2(h_0\rho_0+3h_1\rho_1)\rho_0M_0^{-1}+h_1^2(h_1\rho_1+3h_0\rho_0)\rho_1M_1^{-1});$$

$$\begin{aligned}G_4=&-\frac{1}{6}(h_0^{\ 2}(2P_2h_0\rho_0M_0^{-1}+3h_1(\rho_0M_0^{-1}+\rho_1M_1^{-1}))+\\&+h_1^2(2h_1\rho_1M_1^{-1}+3P_2h_0(\rho_0M_0^{-1}+\rho_1M_1^{-1})));\end{aligned}\quad (6)$$

$$G_5=\frac{1}{6}M_1^{-2}(h_0^2(P_2h_0+3h_1)+h_1^2(h_1+3P_2h_0));$$

$$\begin{aligned}G_6=&\frac{1}{120}(h_0^5P_2\rho_0^2M_0^{-2}(10\rho_1M_1^{-1}+\rho_0M_0^{-1})+h_1^5\rho_1M_1^{-1}(10\rho_0M_0^{-1}+\rho_1M_1^{-1})+\\&+5h_0h_1\rho_0\rho_1M_0^{-1}M_1^{-1}(h_0^3\rho_0M_0^{-1}(3-3D_0-D_0^2)-h_1^3P_2\rho_1M_1^{-1}(3-3D_1-D_1^2)));\end{aligned}$$

$$\begin{aligned}G_7=&\frac{1}{120}(-13(h_0^5P_2\rho_0^2M_0^{-2}+h_1^5\rho_1^2M_1^{-2})+20(h_0^5P_2+h_1^5)\rho_0\rho_1M_0^{-1}M_1^{-1}-\\&-5h_0h_1(h_0^3\rho_0M_0^{-1}((3-3D_0-D_0^2)\rho_0M_0^{-1}-(D_0-4)\rho_1M_1^{-1})+\\&+h_1^3P_2\rho_1M_1^{-1}((3-3D_1-D_1^2)\rho_1M_1^{-1}-(D_0-4)\rho_1M_1^{-1}\rho_0M_0^{-1})));\end{aligned}\quad (6)$$

$$\begin{aligned}G_8=&(23(h_0^5P_2\rho_0^2M_0^{-1}+h_1^5\rho_1^2M_1^{-2})+10(h_0^5P_2\rho_1M_1^{-1}+h_1^5\rho_0M_0^{-1})+\\&+5h_0h_1(h_0^3(\rho_1M_1^{-1}-(D_0-4)\rho_0M_0^{-1})+h_1^4(\rho_0M_0^{-1}-(D_1-4)\rho_1M_1^{-1})));\end{aligned}$$

$$F_2(x,y,t)=P_2(N_0^{-1}(\frac{\partial^2 f_{xz}^{(0)}}{\partial x^2}-\frac{\partial^2 f_{yz}^{(0)}}{\partial y^2})+N_1^{-1}(\frac{\partial^2 f_{xz}^{(1)}}{\partial x^2}-\frac{\partial^2 f_{yz}^{(1)}}{\partial y^2}))+$$

$$+\frac{1}{2}(P_2h_1^2\rho_1M_1^{-1}(N_0^{-1}\frac{\partial^2 f_{xz}^{(0)}}{\partial x^2}-\frac{\partial^2 f_{yz}^{(0)}}{\partial y^2})-$$
$$-h_0^2\rho_0M_0^{-1}(N_1^{-1}\frac{\partial^2 f_{xz}^{(1)}}{\partial x^2}-\frac{\partial^2 f_{yz}^{(1)}}{\partial y^2}))\frac{\partial^2}{\partial t^2}-$$
$$-\frac{1}{2}(P_2h_1^2(N_0^{-1}\frac{\partial^2 f_{xz}^{(0)}}{\partial x^2}-\frac{\partial^2 f_{yz}^{(0)}}{\partial y^2})-$$
$$-h_0^2(N_1^{-1}\frac{\partial^2 f_{xz}^{(1)}}{\partial x^2}-\frac{\partial^2 f_{yz}^{(1)}}{\partial y^2}))\frac{\partial^2}{\partial x^2}.$$

Despite the fact that equation (1) is approximate, it is rather complicated. Operators (2) contain all the parameters and operators that characterize both the mechanical and rheological properties of the material of a piecewise homogeneous plate and its geometric dimensions.

Approximate equation (1) is simplified in special cases when solving specific vibration problems. For example, operators (2) are greatly simplified when the Poisson's ratios of both components are constant, or when the thicknesses of both components are equal, and so on.

For example , if $h_0 = h_1$ and $\nu_0 = \nu_1$ then the operators Q_j in (2) have the form:

$$Q_1 = M_1^{-2}h_0^2(\rho_0+\rho_1)^2;$$
$$Q_2 = -2M_1^{-2}h_0^2(2D_0(P_2+1)(\rho_0+\rho_1)+(P_2+1)(2\rho_0-D_0(\rho_0-\rho_1)));$$
$$Q_3 = 4(P_2-1)h_0^{\ 2}D_0(3P_2+1);$$
$$Q_4 = -\frac{1}{6}M_1^{-2}h_0^{\ 4}(2-D_0)(\rho_0M_0^{-1}(3\rho_1^2+\rho_0(\rho_0+4\rho_1))+$$
$$+\rho_1M_1^{-1}(3\rho_0^2+\rho_1(\rho_1+4\rho_0)));$$
$$Q_5 = -\frac{1}{6}h_0^{\ 4}(P_2\rho_0^2M_0^{-2}(4D_0(4-D_0)+P_2(8D_0(1-D_0)+5)+$$
$$+(P_2-1)(12-6D_0+D_0^2))+2\rho_0\rho_1M_0^{-1}M_1^{-1}(2(6D_0+P_2^2(2+5D_0)+$$
$$+P_2(2+9D_0-D_0^2))+(P_2-1)P_2(2-3D_0+D_0^2)+D_0(1+D_0))+ \quad (7)$$
$$+\rho_1^2M_1^{-2}(8(1+D_0-D_0^2)+4P_2D_0(4-D_0)+(P_2-1)D_0(2-D_0)));$$
$$Q_6 = \frac{1}{3}h_0^{\ 2}(\rho_0M_0^{-1}(4P_2D_0(2+5P_2-3D_0(P_2-1))+(P_2-1)(P_2(20-8D_0-13D_0^2)+$$
$$+6D_0(1-D_0)))+\rho_1M_1^{-1}D_0(4(4+D_0)+4P_2(4+2P_2+5D_0)+$$
$$+17(P_2-1)(D_0+2P_2(1-D_0))));$$
$$Q_7 = \frac{4}{3}h_0^{\ 4}D_0(D_0(4-15P_2-5P_0^2)+(P_2-1)(1-13P_2));$$

The sixth order operator in equation (1) can also be represented as a product of second and fourth order operators if the plate is elastic and the coefficients Q_j are related by

$$Q_2 \cdot Q_4 \cdot Q_7 = Q_1 \cdot Q_5 \cdot Q_7 + Q_3 \cdot Q_4 \cdot Q_6. \qquad (8)$$

For a two-layer elastic plate with given parameters of its components, relation (7) gives a 10th order algebraic equation with respect to the ratio h_2 / h_1, while the sixth order operator in (1) can be represented as a product of two lower order operators

$$\left(A_1 \frac{\partial^2}{\partial t^2} + A_2 \frac{\partial^2}{\partial x^2} \right) \cdot \left(A_3 \frac{\partial^2}{\partial t^2} + A_4 \frac{\partial^2}{\partial x^2} + A_5 \frac{\partial^4}{\partial t^4} + A_1 \frac{\partial^4}{\partial x^4} \right) (W) = 0,$$

if the coefficients Q_j and A_3 bound by dependencies

$$Q_4 = A_1 A_5; \quad Q_5 = A_2 A_5; \quad Q_6 = A_1 A_6; \quad Q_7 = A_2 A_6;$$

CONCLUSIONS

1.The study of vibrations of piecewise-homogeneous plates in the exact three-dimensional formulation makes it possible to derive the general and approximate equations of vibration of such plates based on them without resorting to any hypotheses.

2. It is shown that the simplest approximate equation for the vibration of a two-layer plate is the sixth-order equation in derivatives, which describes its longitudinal-transverse vibration.

3. For an elastic two-layer plate, the sixth-order operator decomposes into the product of second-order operators - longitudinal and fourth order - transverse oscillation, if the thicknesses of the plate components satisfy the derived equation containing the parameters of these components.

4. Formulas are obtained for determining displacements and stresses in terms of the desired functions at any point of a two-layer plate.

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