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Eleven Proof of the Cosinus Form of Two Angle Differences

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ANNOTATION

This paper presents eleven different methods for proving addition formulas for trigonometric functions. The first methods of proof are given by the seventh trigonometric function defined in Mirzo Ulugbek's work. Other methods of proof are based on the equality of triangles, formulas for calculating the area of a triangle, the distance between two points, definitions of trigonometric functions, basic trigonometric identities, and operations on vectors.

KEYWORDS: *Trigonometric functions, saxm(x), circle, basic trigonometric identity, right triangle, Pythagorean theorem, equilateral triangle, surface of a triangle, distance between two points, sum of vectors, scalar product.*

Introduction. Educating the younger generation to be creative, free, independent, critical and logical thinkers, able to compare, analyze and synthesize remains an urgent task. The development of skills to look at a problem from a different perspective and choose the optimal one from among the possible ways to solve it is effective in mastering the variety and historical evidence of affirmations and learning to solve problems in several ways.

Literature review. In this paper, the proof of the formula for the cosine of two angular differences (sum) using the function "saxm(x)" introduced by Mirzo Ulugbek is given in [1], [2], [3], [4], [5] [6], [7], [8], [9] the equations of triangles, formulas for calculating the area of a triangle, the distance between two points, definitions of trigonometric functions, proofs based on operations on basic trigonometric identities and vectors, as well as our colleague the proof presented by and our own proof variant. Mirzo Ulugbek's work "Ziji jadidi Koragoniy" (Koragoniy new astronomical table) was written in the Arabic alphabet in Persian in Samarkand in 1437, and later translated into Arabic and Turkish. It was first published in Europe in 1648 in England. One of the oldest copies written in Persian is kept at the Institute of Oriental Studies of the Academy of Sciences of Uzbekistan. The work consists of two parts, the first part of the introduction and the second part of the table of 1018 stars, the conditions of which are given in an elliptical system. The introduction itself is divided into four parts:

1.The methods of calculating the year of the peoples of the East are described. 2. Applied astronomy, which deals with determining the coordinates of celestial bodies, determining the angle between the ellipse and the equator, drawing meridian lines, determining the azimuth of the qibla, determining the length of the year and making differential measurements in the celestial sphere. 3. The apparent motion of the planets is explained on the basis of a geocentric system. 4. Horoscope – provides information about astrology. In the second section, in addition to the table of coordinates of the location of stars, there are also tables of values of sine and tangent, calculated using new methods developed by Samarkand scientists. Mirzo Ulugbek in his work called the sine "jayb", the arcinus "jaybu makus" (inverse sinus), and the 1st cosx as a separate "sahm (x)" as the seventh trigonometric function.

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Research methodology. The study of the literature is based on generalization, comparison and logical analysis of experiments and operations on equality of triangles, formulas for calculating the area of a triangle, distance between two points, definitions of trigonometric functions, basic trigonometric identities and vectors.

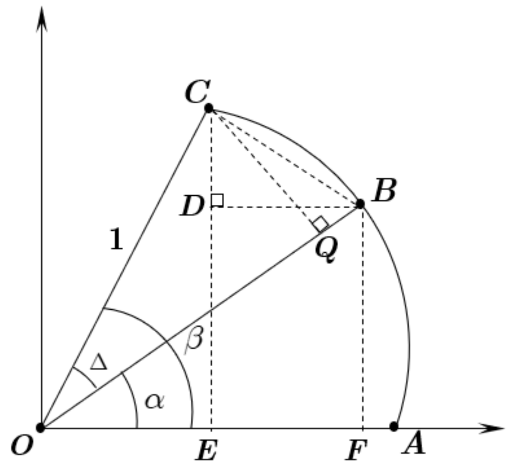
Analysis and results. Proof of theorems in several different ways plays an important role in developing students' thinking skills and adaptability to different situations. From a methodological point of view, a multi-pronged approach to solving a single problem can be effective in mastering new topics. Comparison of equations of triangles, formulas for calculating the area of a triangle, distances between two points, definitions of trigonometric functions, basic trigonometric identities, and operations based on vectors in proof of addition formulas accelerates students' mastery. The student chooses the most optimal method of proof for himself and tries to find the ways to prove the theorem independently.

Theorem. For any α and β, the following equation holds:

$$\cos(\alpha-\beta) = cos\alpha \cdot cos\beta + sin\alpha \cdot sin\beta \qquad (1)$$

The first proof ([2]). $\breve{AB}=\alpha, \breve{AC}=\beta,\ \breve{BC}=\beta-\alpha=\Delta,\ OA=OB=OC=1,$

1) $BD \perp CE,\ CQ \perp OB,\ \Delta CDB=\Delta CQB.$ (1-look at the picture)



1-picture

2) From the Pythagorean theorem $|BC|^2=|QC|^2+|QB|^2=|CD|^2+|DB|^2$ or $DE=BF, DB=EF$, $(CQ)^2+(OB-OQ)^2=(CE-DE)^2+(OF-OE)^2$

$$\frac{CQ}{1}=sin\Delta,\ \frac{OQ}{1}=cos\Delta,\ \frac{CE}{1}=sin\beta,\ \ \frac{BF}{1}=sin\alpha,\frac{OF}{1}=cos\alpha,\ \frac{OE}{1}=cos\beta,$$

$$sin^2\Delta+(1-cos\Delta)^2=(sin\beta-sin\alpha)^2+(cos\alpha-cos\beta)^2,$$

$$sin^2\Delta+1-2cos\Delta+cos^2\Delta= sin^2\beta-2sin\alpha\cdot sin\beta+sin^2\alpha+$$

$$cos^2\alpha-2cos\alpha\cdot cos\beta+cos^2\beta, 2-2\cdot cos\Delta= 2-sin\beta\cdot sin\alpha-cos\alpha\cdot cos\beta\ ,$$

$$\Delta= \beta-\alpha\ , \quad cos\Delta= cos\alpha\cdot cos\beta+sin\alpha\cdot sin\beta.$$

1-lemma. For any α and β, the following formula is appropriate:

$$\sin(\alpha+\beta)=sin\alpha\cdot cos\beta+cos\alpha\cdot sin\beta \qquad (2)$$

Proof: $\sin(\alpha+\beta)$ we write:

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$$\sin(\alpha+\beta) = \cos\left(\frac{\pi}{2} - (\alpha+\beta)\right) = \cos(\left(\frac{\pi}{2} - \alpha\right) - \beta).$$

If we use formula (1) for the last equation, we get formula (2).

2-lemma. For any α and β, the following formula is appropriate:

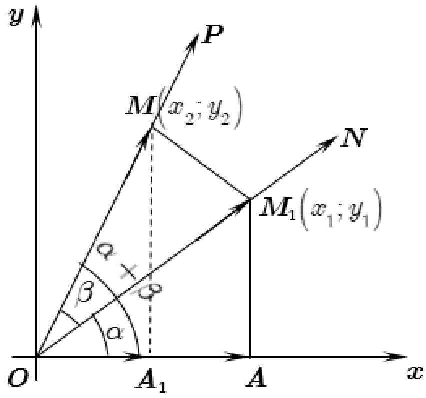
$$\cos(\alpha + \beta) = \cos\alpha \cdot cos\beta - sin\alpha \cdot sin\beta \qquad (3)$$

$$\sin(\alpha - \beta) = \sin\alpha \cdot cos\beta - cos\alpha \cdot sin\beta \ . \qquad (4)$$

Proof: Formulas (3) and (4) are obtained by substituting β for (-β) in formulas (1) and (2) and using even and odd functions of sine and cosine functions.

The second proof ([3]**).**

We use projections of vectors (2-look at the picture):



2-picture

1) $\left|\overrightarrow{OM}\right| = 1, \quad MM_1 \perp ON,$ (1) 2) $\overrightarrow{OM} = \overrightarrow{OM_1} + \overrightarrow{M_1M}$ (2) $\overrightarrow{OM_1} = \{x_1, y_1\}$, $\overrightarrow{M_1M} = \{x_2, y_2\}$,

$\overrightarrow{OM} = \{x_1 + x_2, \ y_1 + y_2\}$, 3) $\Delta OAM \Rightarrow \cos(\alpha+\beta) = \frac{x_1+x_2}{|\overrightarrow{OM}|} = x_1 + x_2$ (3)

$x_1 = \left|\overrightarrow{OM_1}\right| \cdot cos\alpha \ ; \qquad x_2 = \left|\overrightarrow{M_1M}\right| \cdot \cos(90^0 + \alpha) = -\left|\overrightarrow{M_1M}\right| \cdot sin\alpha$

4) We set the values of x_1 and x_2 to (3). $\cos(\alpha+\beta) = \left|\overrightarrow{OM_1}\right| \cdot \cos\alpha + \left(-\left|\overrightarrow{M_1M}\right|\right) \cdot sin\alpha = cos\alpha \cdot cos\beta - sin\beta \cdot sin\alpha$;

Similarly, the unit can be done in a circle. $\left|P_\alpha P_\beta\right| = \left|P_0 P_{\alpha+\beta}\right|$,

$y_1 = \left|\overrightarrow{OM_1}\right| \cdot sin\alpha \ ; \ \ y_2 = \left|\overrightarrow{M_1M}\right| \cdot \sin(90^0 + \alpha) = \left|\overrightarrow{M_1M}\right| \cdot cos\alpha,$

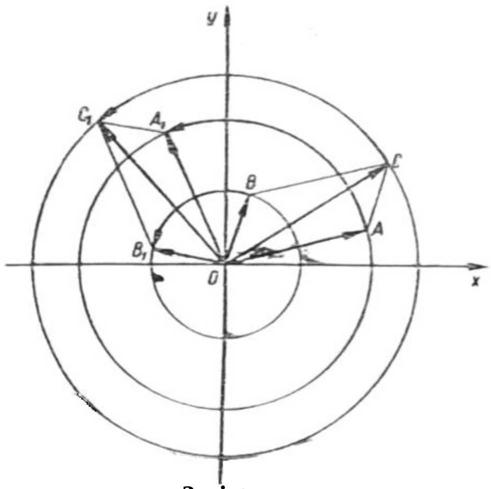
$$\sin(\alpha+\beta) = \left|\overrightarrow{OM_1}\right| \cdot sin\alpha + \left|\overrightarrow{M_1M}\right| \cdot cos\alpha = sin\alpha \cdot cos\beta - sin\beta \cdot cos\alpha;$$

The third proof ([4]**).**

Here are two obvious geometric notes.

1. By turning the vector $k\vec{a}$ at an angle a, we create a vector $R_\alpha(k\vec{a}) = kR_\alpha(\vec{a})$, where k is a multiplier.
2. Turning the sum of vectors $\vec{a} + \vec{b} = \vec{c}$ to an angle a, we obtain a vector $R_\alpha(\vec{c})$ equal to the sum of vectors $R_\alpha(\vec{a})$ and $R_\alpha(\vec{b})$.

To make sure that the second note is correct, we look at the vectors $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, and $\overrightarrow{OC} = \vec{c}$ that satisfy the equation $\vec{a} + \vec{b} = \vec{c}$ The angle R_α reflects the *OABC* parallelogram to the parallelogram $OA_1B_1C_1$ (3-look at the picture). As a result, we obtain the following equations: $R_\alpha(\vec{a}) = \overrightarrow{OA_1}$, $R_\alpha(\vec{b}) = \overrightarrow{OB_1}$, $R_\alpha(\vec{c}) = \overrightarrow{OC_1}$.



3-picture

That is why:

$$R_\alpha(\vec{c}) = R_\alpha(\vec{a} + \vec{b}) = R_\alpha(\vec{a}) + R_\alpha(\vec{b}).$$

Using these notes, $$R_\alpha(\vec{\imath}) = cos\alpha \cdot \vec{\imath} + sin\alpha \cdot \vec{\jmath} \quad (1)$$

in addition to equality

$$R_\alpha(\vec{\jmath}) = -sin\alpha \cdot \vec{\imath} + cos\alpha \cdot \vec{\jmath} \quad (2)$$

prove the formula. Indeed,

$$R_\alpha(\vec{\jmath}) = R_\alpha\left(R_{\frac{\pi}{2}}(\vec{\imath})\right) = R_{\frac{\pi}{2}}(R_\alpha(\vec{\imath})) = R_{\frac{\pi}{2}}(cos\alpha \cdot \vec{\imath} + \ sin\alpha \cdot \vec{\jmath}) =$$

$$= cos\alpha \cdot R_{\frac{\pi}{2}}(\vec{\imath}) + sin\alpha \cdot R_{\frac{\pi}{2}}(\vec{\jmath}) = cos\alpha \cdot \vec{\jmath} + sin\alpha \cdot (-\vec{\imath}) = -sin\alpha \cdot \vec{\imath} + cos\alpha \cdot \vec{\jmath}.$$

Find cos (α + β) of sin (α + β) and α of β represented by the cosines and sine of the angles.

$$\vec{e}_{\alpha+\beta} = cos(\alpha + \beta) \cdot \vec{\imath} + sin(\alpha + \beta) \cdot \vec{\jmath} \quad (3)$$

when they, on the other hand,

$$\vec{e}_{\alpha+\beta} = R_\beta(\vec{e}_\alpha) = R_\beta(cos\alpha \cdot \vec{\imath} + sin\alpha \cdot \vec{\jmath}) = cos\alpha \cdot R_\beta(\vec{\imath}) + sin\alpha \cdot R_\beta(\vec{\jmath})$$

we see. (1) and (2) higher education association:

$$\vec{e}_{\alpha+\beta} = cos\alpha \cdot (cos\beta \cdot \vec{\imath} + sin\beta \cdot \vec{\jmath}) + sin\alpha \cdot (-sin\beta \cdot \vec{\imath} + cos\beta \cdot \vec{\jmath}) =$$

$$= (cos\alpha \cdot cos\beta - sin\alpha \cdot sin\beta) \cdot \vec{\imath} + (sin\alpha \cdot cos\beta - cos\alpha \cdot sin\beta) \cdot \vec{\jmath} \quad (4)$$

comparing equations (3) and (4), we find:

$$\cos(\alpha + \beta) = cos\alpha \cdot cos\beta - sin\alpha \cdot sin\beta;$$

$$\sin(\alpha+\beta) = sin\alpha \cdot cos\beta + cos\alpha \cdot sin\beta. \qquad (5)$$

(5) the cosine and sine of a set of formulas are called formulas. If we use the formulas β and $-\beta$ balance, $\cos(-\beta) = cos\beta$ and $\sin(-\beta) = -sin\beta$ then draw the tables, the tables are as follows:

$$\cos(\alpha-\beta) = cos\alpha \cdot cos\beta + sin\alpha \cdot sin\beta;$$

$$\sin(\alpha-\beta) = sin\alpha \cdot cos\beta - cos\alpha \cdot sin\beta. \qquad (6)$$

(6) The difference between the formulas is called the cosine and sine formulas.

The fourth proof ([5]**).**

Let's look at a trigonometric circle with center O and radius 1 (4-look at the picture). Rotating the starting point $P(1;0)$ around the origin to angles α and β forms points *A* and *B*, respectively. According to the definition of the sine and cosine of an angle in a trigonometric circle, the coordinates of points *A* and *B* are:

$$A(cos\alpha; sin\alpha) \text{ va } B(cos\beta; sin\beta) \qquad (1.1)$$

According to (1.1) we calculate the coordinates of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$:

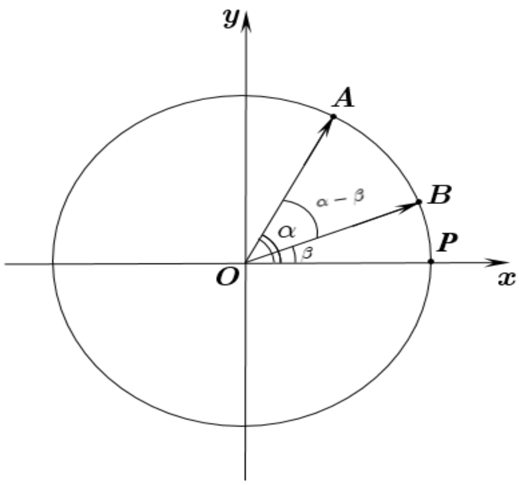
$$\overrightarrow{OA} = (cos\alpha; sin\alpha) \quad \text{va} \quad \overrightarrow{OB} = (cos\beta; sin\beta) \qquad (1.2)$$

Given that the angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is $\alpha-\beta$, the scalar product of these vectors

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = |\overrightarrow{OA}| \cdot |\overrightarrow{OB}| \cdot \cos(\alpha-\beta) \qquad (1.3)$$

will be on the other hand, according to (1.2)

$$\overrightarrow{OA} \cdot \overrightarrow{OB} = \cos\alpha \cdot cos\beta + sin\alpha \cdot sin\beta \qquad (1.4)$$



4-picture

Finally, from the equality of the left sides of (1.3) and (1.4), the equality of the right sides also follows, that is, the formula we are proving is formed:

$$\cos(\alpha-\beta) = \cos\alpha \cdot cos\beta + sin\alpha \cdot sin\beta.$$

The fifth proof ([6]**).**

We use the formula for the distance between two points. Rotating the point $M_0(1;0)$ around the coordinate head to the radian angles α, β, $\alpha+\beta$, $-\alpha$ results in the formation of points M_α, $M_{-\alpha}, M_\beta$ *and* $M_{\alpha+\beta}$, respectively (5-look at the picture).

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1) $|P_{-\alpha}P_\beta| = |P_0 P_{\alpha+\beta}|$, 2) $P_{-\alpha} = M(cos\alpha, -sin\alpha)$, $P_\beta = N(cos\beta; sin\beta)$.

Find the distance between the points,

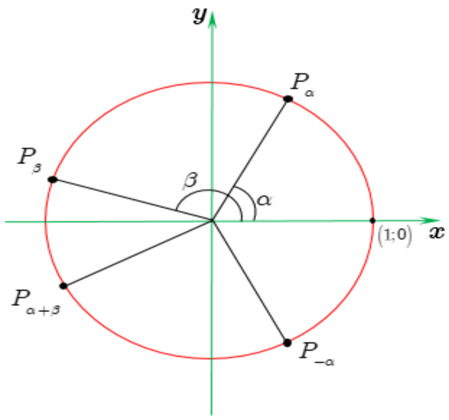
$$d_1(MN) = \sqrt{(cos\alpha - cos\beta)^2 + (-sin\alpha - sin\beta)^2}$$

Similarly, , $P_0 = M(1,0)$ va $P_{\alpha+\beta} = N_1(\cos(\alpha+\beta), \sin(\alpha+\beta))$,

Find the distance between the points,

$$d_1(MN) = \sqrt{(1 - (\cos(\alpha+\beta))^2 + (0 - \sin(\alpha+\beta))^2}$$

In the equation of triangles $d_1 = d_2$



5-picture

$$(cos\alpha - cos\beta)^2 + (sin\alpha + sin\beta)^2 = (1 - \cos(\alpha+\beta))^2 + (0 - \sin(\alpha+\beta))^2$$

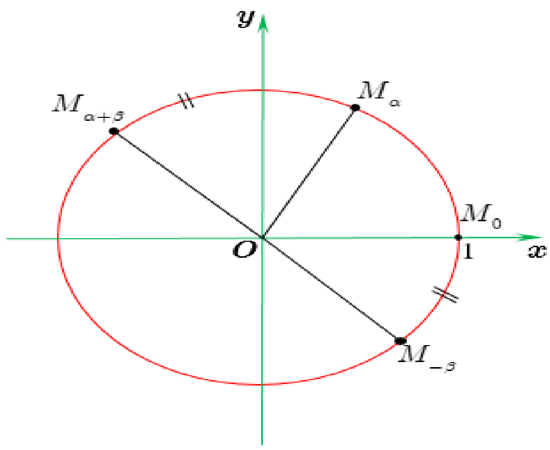
$$cos^2\alpha - 2cos\alpha \cdot cos\beta + cos^2\beta + sin^2\alpha + 2sin\alpha \cdot sin\beta + sin^2\beta$$
$$= 1 - 2\cos(\alpha+\beta) + cos^2(\alpha+\beta) + sin^2(\alpha+\beta)$$

$$2 - 2(\cos\alpha \cdot cos\beta - sin\alpha \cdot sin\beta) = 2 - 2\cos(\alpha+\beta)$$

$$\cos(\alpha+\beta) = \cos\alpha \cdot cos\beta - sin\alpha \cdot sin\beta.$$

The sixth proof ([7]).

Let's use the equation of triangles (6-look at the picture):



6-picture

$\angle M_0 O M_{\alpha+\beta} = \angle M_{-\beta} O M_{\alpha}$, $M_{\alpha}(cos\alpha; sin\alpha)$, $M_{-\beta}(\cos(-\beta); \sin(-\beta)), M_{\alpha+\beta}(\cos(\alpha+\beta); \sin(\alpha+\beta))$. From equilateral triangles, their bases are also equal. $(M_0 M_{\alpha+\beta})^2 = (M_{-\beta} M_{\alpha})^2 \Rightarrow (1-\cos(\alpha+\beta))^2 + (\sin(\alpha+\beta))^2 = (\cos(-\beta) - cos\alpha)^2 + (\sin(-\beta) - sin\alpha)^2$, $2 - 2\cos(\alpha+\beta) = 2 - 2cos\alpha \cdot cos\beta + 2sin\alpha \cdot sin\beta$.

$$\cos(\alpha+\beta) = \cos\alpha \cdot cos\beta + sin\alpha \cdot sin\beta.$$

$$\cos(\alpha + (-\beta)) = \cos\alpha \cdot cos\beta + sin\alpha \cdot sin\beta.$$

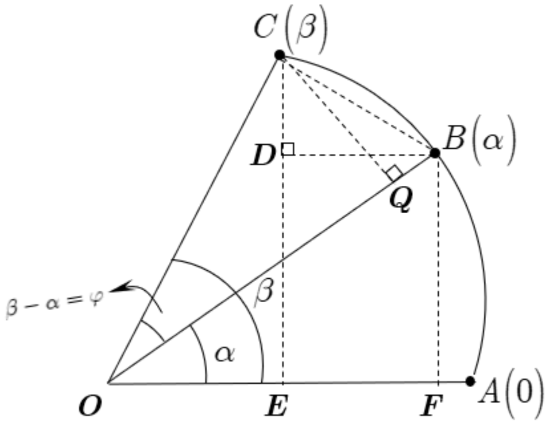
The seventh proof ([2]**).**

$BD \perp CE, CQ \perp OB,\ DE = BF;\ DB = EF = OF - OE = cos\alpha - cos\beta$,

$QB = OB - OQ = 1 - cos\varphi,\ CQ = sin\varphi$, $CD = CE - BF = sin\beta - sin\alpha$,

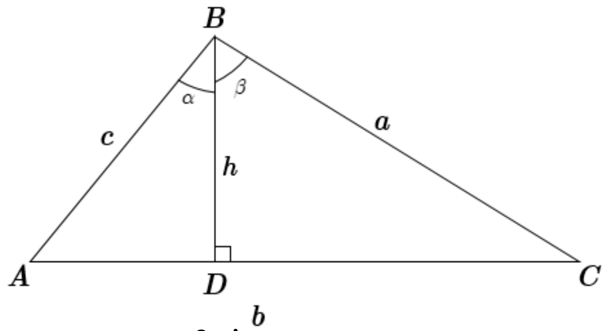
$OA = OB = OC = R = 1$, CDB and CQB right triangles have a common hypotenuse (7-look at the picture).

From the Pythagorean theorem, $BC^2 = CQ^2 + QB^2 = CD^2 + BD^2$ or $sin^2\varphi + (1 - cos\varphi)^2 = (sin\beta - sin\alpha)^2 + (cos\alpha - cos\beta)^2$, $\Rightarrow$ $\cos(\beta - \alpha) = \cos\alpha \cdot cos\beta + sin\alpha \cdot sin\beta$ arises.



7-picture

The eighth proof ([8]**).**



8-picture

Proof by the surfaces of a triangle. (8-look at the picture).

$$sin(\alpha+\beta) = sin\,\alpha \cdot cos\beta + cos\alpha \cdot sin\beta$$

1) $BD \perp AC$, $\frac{h}{a} = cos\beta$, $\frac{h}{c} = cos\alpha$, $S_{\Delta ABC} = \frac{1}{2}ac \cdot \sin(\alpha+\beta)$, $S_{\Delta ABC} = S_{\Delta ABD} + S_{\Delta BDC}$, $S_{\Delta ABD} = \frac{1}{2}c \cdot h \cdot \sin\alpha = \frac{1}{2}c \cdot \sin\alpha \cdot a \cdot cos\beta = \frac{1}{2}ac \cdot \sin\alpha \cdot cos\beta$, $S_{\Delta CBD} = \frac{1}{2}a \cdot h \cdot \sin\beta = \frac{1}{2}a \cdot$

$\sin\beta\cdot c\cdot cos\alpha=\frac{1}{2}ac\cdot\sin\beta\cdot cos\alpha$,

$\frac{1}{2}ac\cdot\sin(\alpha+\beta)=\frac{1}{2}ac\cdot\sin\alpha\cdot cos\beta+\frac{1}{2}ac\cdot\sin\beta\cdot cos\alpha$, divide both sides of the equation by $\frac{1}{2}ac$, $\sin(\alpha+\beta)=\sin\alpha\cdot cos\beta+\sin\beta\cdot cos\alpha$.

The ninth proof ([9]**)**

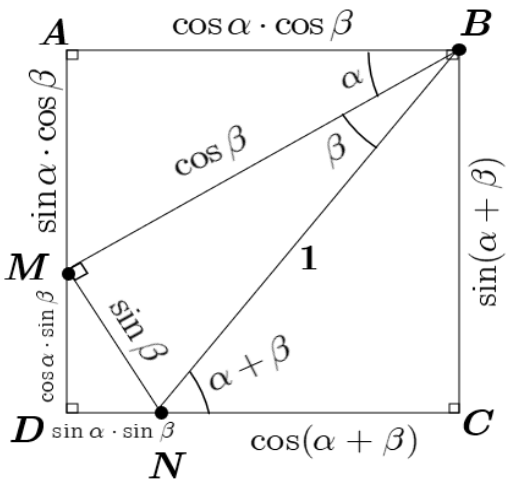
Draw the straight lines *BM* and *BN* from the end *B* of the rectangle $ABCD$ to the sides *AD* and *DC* so that $BM\perp MN$

(9-look at the picture). We enter the following definitions. $\angle ABM=\alpha$, $\angle MBN=\beta$, $BN=1$. In this case, using the definitions of the sine and cosine of the acute angle in a right triangle, we find the catheters in the right triangles BMN, ABM, BCN, DMN. $MN=sin\beta$, $BM=cos\beta$, $CN=\cos(\alpha+\beta)$, $BC=\sin(\alpha+\beta)$,

$AB=cos\alpha\cdot cos\beta$, $AM=sin\alpha\cdot cos\beta$, $DM=cos\alpha\cdot sin\beta$, $DN=sin\alpha\cdot sin\beta$. $(*)$

Finally, given the equations $AM+MD=BC$ and $DN+NC=AB$, according to (*)

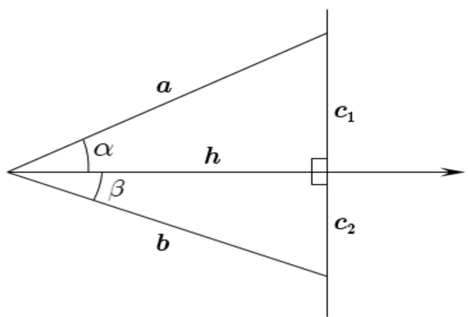
$\cos(\alpha+\beta)=\cos\alpha\cdot cos\beta-sin\alpha\cdot sin\beta$, $\sin(\alpha+\beta)=sin\alpha\cdot cos\beta+cos\alpha\cdot sin\beta$.



9-picture

Tenth proof.(Prof. Yaxshimurodov A. presented)

1) $c_1+c_2=c$, a) $a^2=h^2+c_1^2$, $\Rightarrow$ $a^2-c_1^2=h^2$, $b^2=h^2+c_2^2$, $\Rightarrow$ $a^2-c_2^2=h^2$ b) $c^2=a^2+b^2-2ab\cdot\cos(\alpha+\beta)$, $b)\Rightarrow\cos(\alpha+\beta)=\frac{a^2+b^2-(c_1+c_2)^2}{2ab}=\frac{(a^2-c_1^2)+(b^2-c_2^2)-2c_1c_2}{2ab}=a)=\frac{2h^2-2c_1c_2}{2ab}=\frac{h^2}{ab}-\frac{c_1c_2}{ab}=\frac{h}{a}\cdot\frac{h}{b}-\frac{c_1}{a}\cdot\frac{c_2}{b}=$(10-look at the picture)



10 - picture

$$= cos\alpha \cdot cos\beta - sin\alpha \cdot sin\beta \ ;$$

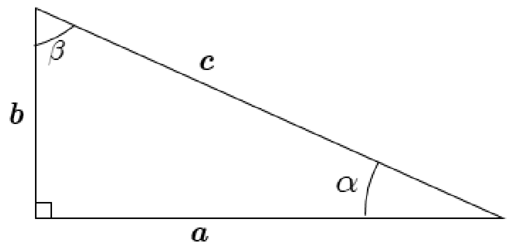
$$\left(\frac{h}{a} = cos\alpha, \ \ \frac{h}{b} = cos\beta, \qquad \frac{c_1}{a} = sin\alpha, \ \ \frac{c_2}{b} = sin\beta \right)$$

2) $S = \frac{1}{2}\text{ab} \cdot \sin(\alpha+\beta)$, $\sin(\alpha+\beta) = \frac{2S}{ab}$, $\sin(\alpha+\beta) = \frac{2(S_1+S_2)}{ab} = \frac{2(\frac{c_1 h}{2}+\frac{c_2 h}{2})}{ab} =$

$$= \frac{c_1 h + c_2 h}{2} = \frac{c_1}{a} \cdot \frac{h}{b} - \frac{h}{a} \cdot \frac{c_2}{b} = sin\alpha \cdot cos\beta + cos\alpha \cdot sin\beta \ .$$

The eleventh proof (Of the authors).

Proof of a right triangle using the Pythagorean theorem.



11-picture

$\forall\ \alpha,\beta \in \left(0;\frac{\pi}{2}\right)$ 1) As you know, $\frac{b}{c} = sin\alpha$, $\frac{a}{c} = sin\beta$, $\frac{a}{c} = cos\alpha$, $\frac{b}{c} = cos\beta$, $\frac{b}{a} = tg\alpha$, $\frac{a}{b} = ctg\alpha$, $\frac{b}{a} = ctg\beta$, $\frac{a}{b} = tg\beta$. (1) (11-look at the picture) 2) $S = \frac{1}{2}\text{a} \cdot \text{c} \cdot \sin\alpha$, $S = \frac{1}{2}\text{b} \cdot \text{c} \cdot \sin\beta$, $\Rightarrow S^2 = \frac{1}{4}\text{a} \cdot \text{b} \cdot c^2 \cdot \sin\alpha \cdot sin\beta$, (2) 3) $\alpha + \beta = \frac{\pi}{2}$, $\sin(\alpha+\beta) = 1$ (3) 4) $S = \frac{ab}{2} \Leftrightarrow ab = 2S$ (4)

5) (2) and (4) $\Rightarrow (\frac{ab}{2})^2 = \frac{1}{4} a \cdot b \cdot c^2 \cdot sin\,\alpha \cdot sin\beta \ \Rightarrow \ a \cdot b = c^2 \cdot sin\,\alpha \cdot sin\beta$ (5)

6) (4) and (5) $\Rightarrow \ 2S = c^2 \cdot sin\,\alpha \cdot sin\beta \ \Leftrightarrow \ \frac{2 \cdot a \cdot b \cdot sin 90^0}{2} = c^2 \cdot sin\,\alpha \cdot sin\beta$ (3) $\Rightarrow$

$$a \cdot b \cdot sin(\alpha+\beta) = c^2 \cdot sin\,\alpha \cdot sin\beta, \ sin(\alpha+\beta) = \frac{c^2}{ab} \cdot sin\alpha \cdot sin\beta =$$

$$= \frac{a^2+b^2}{ab} \cdot sin\alpha \cdot sin\beta \ = \left(\frac{a}{b} + \frac{b}{a}\right) \cdot sin\alpha \cdot sin\beta = (ctg\alpha + ctg\beta) \cdot sin\alpha \cdot sin\beta =$$

$$= ctg\alpha \cdot sin\alpha \cdot sin\beta + ctg\beta \cdot sin\alpha \cdot sin\beta = sin\alpha \cdot cos\beta + sin\beta \cdot cosa =$$

$$= \frac{cos\alpha}{sin\alpha} \cdot sin\alpha \cdot sin\beta + \frac{cos\beta}{sin\beta} \cdot sin\alpha \cdot sin\beta = cos\alpha \cdot sin\beta + sin\alpha \cdot cos\beta \ \ \Rightarrow$$

$sin(\alpha+\beta) = cos\alpha \cdot sin\beta + sin\alpha \cdot cos\beta$, $sin(\alpha-\beta) = sin\alpha \cdot cos\beta - cos\alpha \cdot sin\beta$;

Conclusions and recommendations. Proof of a number of theorems plays an important role in developing students’ thinking skills and adaptability to different situations. Proof of assertion in different ways and solving problems in several ways will accelerate students’ mastery. As a result, students develop the skills of comparison, synthesis and analysis, critical and creative thinking. This will increase students’ interest in mathematics.

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