"The Important Role of Gears in Mechanical Engineering"

Nazarova Nafisa Ilhomovna,
work distributor, Navoi Machine-Building Plant, Uzbekistan

Abstract

Nowadays, the basic requirements of gear transmissions are not limited to resistance and reliability, but often include good efficiency and low vibration and noise emissions. This article investigates the role of tooth flank micro-geometry in fulfilling these needs. A non-linear finite element approach has been conceived and exploited to investigate in detail the influence of the shape of profile modifications (PMs) on transmission error, root stress, and contact pressure. In this approach, harmonic drive gears are widely used in space applications, robotics, and precision positioning systems because of their attractive attributes including near-zero backlash, high speed reduction ratio, compact size, and small weight. On the other hand, they possess an inherent periodic positioning error known as kinematic error responsible for transmission performance degradation. No definite understanding of the mechanism of kinematic error as well as its characterization is available in the literature. The numerical results are first assessed by comparison with experimental measurements and then a comparison of contact and bending stresses of the same gear with long linear and long circular PMs is presented and discussed. The results of these comparisons show that the optimal amount of PMs is not independent of PM shape; hence, the procedures used to design linear PMs cannot be directly applied to the design of non-linear PMs.

Keywords
gear, gear micro-geometry, profile modifications, transmission error, noise, finite-element method, gear bending and contact stress

Introduction

In modern industry, the designer is commonly restricted by the requirement that mechanical elements, like wheels, gears, bearings, cams, etc., should carry high loads at high speed with both size and weight kept to a minimum. For such applications, predicting component operational failure becomes crucial to ensure an adequate design. In the case of gears, two kinds of teeth damage can occur under repeated loading due to fatigue; namely the pitting of gear teeth flanks and tooth fracture in the tooth root. Planetary gear sets are used commonly by automotive and aerospace industries. Typical applications include jet propulsion systems, rotorcraft transmissions, passenger vehicle automatic transmissions and transfer cases and off-highway vehicle gearboxes. Their high-power-density design combined with their kinematic flexibility in achieving different speed ratios make planetary gears sets often preferable to counter-shaft gear reduction systems. As planetary gear sets possess unique kinematicand geometricproperties, they require specialized design knowledge [1]. One type of the key parameters, the rim thickness of the gears, must be defined carefully by the designer in order to meet certain design objectives regarding power density, planet load sharing, noise and durability. From the power density point of view, the rim of the each gear forming the planetary gear set must be as thin as possible in order to minimize mass. Besides reducing mass, added gear flexibility through reduced rim thickness was shown to reduce the influence of a number of internal gear and carrier errors, and piloting inaccuracies [2]. In addition, it was also reported that a flexible internal gear helps improve the load sharing amongst the planets when a number of manufacturing and assembly related gear and carrier errors are present [3–6]. Many of these effects of flexible gear rims were quantified under quasi-static conditions in the absence of any dynamic effects. The effect of rim thickness on gear stresses attracted significant attention in the past. A number of theoretical studies [7–14] modelled mostly a segment of spur gear with a thin rim. In these studies, the gear segment was typically constrained using certain boundary conditions at the cut ends and a point load along the line of action was applied to a single tooth in order to simulate the forces imposed on a sun or an internal gear by the mating planet. This segment of the gear was modelled by using the conventional finite element (FE) method with the same boundary conditions applied in order to simulate the actual support conditions. While these models were instrumental in qualitatively describing the influence of the rim thickness on the bending stresses of an internal gear, they were not fully capable of describing the behavior observed in a number of experiments on this subject matter [12,15–18]. The accuracy of the stress predictions were strongly dependent on the suitability of the conventional FEM meshes to simulate the tooth, the boundary conditions imposed to represent the actual support conditions, and the assumption that a point load can fully describe the actual loads on planet mesh. Since large portions of the internal gear and the other gears (planets and the sun gear) are left out of these models, it was not possible to investigate the effect of internal gear rim thickness on the overall behavior of the planetary gear set including its influence on the stresses of planets and the sun gear and the load sharing amongst the planets. Similarly, an accurate prediction of the shape and the amount of gear deflections was also not possible for the same reasons. Two recent studies by the first author [2,6] employed a non-linear deformable-body model of an entire planetary gear set to investigate the impact of rim flexibilities, especially of the internal gear, on gear stresses and planet load sharing under static conditions. These studies indicate that reducing rim thickness of the gears improve functionality of the gear set by minimizing the adverse effects of gear and carrier manufacturing errors and by improving the planet load-sharing characteristics under quasi-static conditions. However, these benefits come at the expense of increased gear stresses. The practical design question of how thin gear rim thicknesses can be without any durability problems is not possible to answer based on these staticanalyses alone. It is expected that behavior of the planetary gear set changes under dynamic conditions as the system flexibility is increased, potentially increasing gear stresses to a certain extent.

Vehicle differential hypoid gears are usually subjected to varying load-speed conditions. Key concerns are transmission efficiency, refinement of Noise, Vibration and Harshness (NVH), and mitigating wear/fatigue. Multi-physics models are essential tools when investigating such multi-purpose integrated studies, because there are strong interactions between gear dynamics and contact tribology. This is mainly through generated conjunctional friction between the meshing teeth pairs. Friction is regarded as a major source of power loss in an otherwise lightly damped power train system. It consumes some of the excess engine order vibration energy, which is the underlying cause of various drive train NVH phenomena, such as transmission rattle [1] and axle whine [2]. Thus, friction consumes some energy and improves upon the lightly damped nature of the powertrain. Dynamics of gear pairs have been extensively studied, particularly for parallel axis transmissions [3-5]. There are fewer investigations of non-parallel axis gears, such as hypoid and bevel gears. This is because of the complexity of contact kinematics and meshing characteristics.

Abe et al [8] carried out experiments to show that that axle gear noise could be reduced by modifying the prevailing vibration mode with the addition of an inertial disk. This can be mounted onto either side of the final drive flanges. Another experimental method was proposed by Hirasaka et al [9] to study the body and driveline sensitivity to the transmission error of an axle hypoid gear pair. It was found that the dynamic mesh force was affected by the torsional vibration characteristics of the driveline system. A dynamic model of a hypoid gear set was developed by Donley et al [10], where the mesh point and line-of-action were considered as time invariant. More recently, hypoid gear kinematic models, based on the exact teeth geometry have been proposed [11-13] in order to study the gear pair dynamics with transmission error excitation and Non-Linear Time Variant (NLTV) mesh characteristics. In another work, an NLTV dynamic model of a hypoid gear pair with mesh parameters, represented by a sinusoidal form, was used to investigate the system response [14]. A multi-point mesh model was developed by Wang [15], which was used to analyse the hypoid gear dynamics. In all the above investigations, the time-dependent teeth mesh parameters were expressed in the form of either fundamental harmonics or by inclusion of a few harmonic orders.

1.1. Objectives and scope

The main objective of this study is to investigate the dynamic effects on gear stresses as a function of gear rim thickness parameters and the number of planets in the system. A deformablebody dynamicmodel will be used to simulate a typical automotive automatictransmission planetary unit. The model includes all of the gears of the planetary gear set in their deformable form, addressing the shortcomings of the previous simplified models cited above. A new rim thickness parameter will be introduced that takes into account the size of the gears. The model will be used to quantify the impact of the gear rim flexibilities on dynamic gear stresses. The relationship between the bending modes of the gears and the number of planets in the system will also be demonstrated quantitatively.

2. Deformable-body dynamic model

An analysis of planetary gear sets using conventional FE packages presents a number of major challenges stemming from the geometric, kinematic and loading characteristics of this application. The width of a typical gear contact zone is at least an order of magnitude smaller than the other gear dimensions, requiring a very refined mesh near the contact. The computational time required by such a fine FE model is often overwhelming [19] even under static conditions. In addition, the level of geometric accuracy required from a gear contact analysis is so high that a conventional finite element approach fails to deliver. Finally, there are major difficulties in generating an optimal mesh that is capable of modelling the stress gradients in the critical regions, especially at the tooth root, while minimizing the total number of degrees of freedom of the entire model. The contact model employed in this study overcomes such difficulties by using FEM and surface integral methods in conjunction. The details of this previously developed model can be found in a paper by Vijayakar [20]. The model uses finite element method to compute relative deformations away from the contact zone. Use of finite elements also allows an accurate representation of complex shapes that planetary gears have. The nearfield deformations in the contact zone are computed using semianalytical techniques based on the half-space solution for a concentrated load. This eliminates the need for a very refined mesh along the tooth surfaces. The nearfield semianalytical solution and the farfield finite element solutions are matched at a matching surface. The finite element model implemented here uses separate interpolation schemes for the displacements and coordinates. The tooth surfaces are modelled by elements that have a very large number of co-ordinate nodes, and can therefore accurately represent the involute shape and surface modifications. In the fillet region, the elements have a large number of displacement nodes to correctly capture the steep stress gradients [6]. The model uses a hierarchical representation of the system that is built from many substructures, with each substructure in turn being composed of many substructures. This allows reducing the computational and memory requirements significantly.

2.1. Example system

An example planetary system is chosen here to represent an automotive gear set in the configuration of a final drive unit of a front-wheel-drive automatic transmission. Here, the internal gear is held stationary and the other two central members are given the duties of input and output. Gear rim deflections can be especially important in a final drive planetary gear set since it is the most heavily loaded gear set in the transmission. Also, the internal gear-case interface forms a direct vibration/force path to the case for noise generation. The same configuration applies to other applications such as automotive all-wheel-drive transfer case reduction units and rotorcraft reduction units as well.

Although it is quite difficult to group the mathematical models developed in gear dynamics, the following classification seems appropriate.

(1) Simple Dynamic Factor Models. This group includes most of the early studies in which a dynamic factor that can be used in gear root stress formulae is determined. These

studies include empirical and semi-empirical approaches as well as recent dynamic models constructed just for the determination of a dynamic factor.

(2) Models with Tooth Compliance. There is a very large number of studies that include only the tooth stiffness as the potential energy storing element in the system. That is, the flexibility (torsional and/or transverse) of shafts, bearings, etc., are all neglected.

In such studies, the system is usually modeled as a single degree of freedom spring-mass system. There is an overlap between the first group and this group since such simple models are sometimes developed for the sole purpose of determining the dynamic factor.

(3) Models for Gear Dynamics. Such models include the flexibility of the other elements as well as the tooth compliance. Of particular interest have been the torsional flexibility of shafts and the lateral flexibility of the bearings and shafts along the line of action.

(4) Models for Geared Rotor Dynamics. In some studies, the transverse vibrations of a gear-carrying shaft are considered in two mutually perpendicular directions, thus allowing the shaft to whirl. In such models, the torsional vibration of the system is usually considered.

(5) Models for Torsional Vibrations. The models in the third and fourth groups consider the flexibility of gear teeth by including a constant or time varying mesh stiffness in the model. However, there is also a group of studies in which the flexibility of gear teeth is neglected and a torsional model of a geared system is constructed by using torsionally flexible shafts connected with rigid gears. The studies in this group may be viewed as pure torsional vibration problems, rather than gear dynamic problems

Although the discussion of previous studies will be made according to the above classification, it should be remembered that sometimes it may be very difficult to label a certain study and some models might be considered in more than one group.

In the solution of the system equations, numerical techniques have usually been employed. Although most of the models for which numerical techniques are used are lumped parameter models, some investigators have introduced continuous system or finite

element models. While closed form solutions are given for some simple mathematical models, analog computer solutions have sometimes been preferred for non-linear and more complicated models, particularly in the earlier studies.

In some studies the main objective has been to find the system natural frequencies and mode shapes and, therefore, only free vibration analyses are made. However, usually the dynamic response of the system is analyzed for a defined excitation. In most of the studies, the response of the system to forcing due to gear errors and to parametric excitation due to tooth stiffness variation during the tooth contact cycle is determined.

MODELS FOR GEARED ROTOR DYNAMICS

Pioneer models of this group are those for studying whirling of gear-carrying shafts, rather than the dynamics of the gear itself. Although investigators have studied whirling of disk-carrying shafts for many years, it was not until the I960s that the influence of the constraint imposed by the gear on the whirling of geared shafts was considered in rotor dynamics problems. Seireg [132], in 1966, investigated the whirling of geared shafts experimentally, but he did not develop any model for the analytical study of the problem, although he gave an empirical procedure for predicting the main resonance frequency. The receptance model ofJohnson [83] discussed in the previous section, however, might be considered to be the first attempt to include the constraints imposed by gears in rotor dynamics. The extensive model of Fukuma et al. [92] which is also discussed in the previous section, could also be included in this group, since it is a three-dimensional model and several possible motions of the gear and shaft are considered. However, their model was not developed for rotor dynamics studies, but for gear dynamic problems. In 1975, Mitchell and Mellen [133] presented experimental data indicating the torsional-lateral coupling in a geared high speed rotor system. They pointed out that mathematical models based on uncoupled lateral-torsional effects fail to provide the necessary information for a proper design of high-performance machinery.
Hamad and Seireg [135, 136] investigated the whirling of pinion-gear systems supported on hydrodynamic bearings. First they considered the shaft of the gear to be rigid and ignored the effect of the transmitted load [129]. The model was extended by assuming that the gear rotor was also supported on isoviscous fluid bearings [136]. In this later work, they also considered the transmitted gear load and its effect on whirl amplitude and stability of balanced and unbalanced gears. However, the model developed did not take account of the torsional vibrations. The solution was obtained by using a digital phase-plane method. Daws and Mitchell [137,138] analyzed gear coupled rotors by developing a threedimensional model in which variable mesh stiffness was considered as a time varying three dimensional stiffness tensor. The "force coupling" caused by the interaction of gear deflection and the time varying stiffness was considered in their model which predicted the forced response ofthe system to excitations due to unbalanced rotors and mesh errors. The transfer matrix method extended to branched gear systems was used for the solution. Daws and Mitchell, however, did not consider the "dynamic coupling" terms in their model. Later, Mitchell and David [139] showed that the magnitude of the dynamic coupling terms is potentiaily as large as the magnitude ofthe linear terms that are included in most rotor analyses. David [140] investigated the dynamic coupling in non-linear geared rotor systems. He improved the model of Daws, in particular by including the second order coupling terms. It was found that the inclusion of dynamic coupling effects changed the predicted response amplitudes of a trial system by four' to eight orders of magnitude at some frequencies. It was also shown that th~ dynamic coupling is capable of producing system responses of the same magnitude as the unbalanced response. With the same model, David and Mitchell [140,141] also studied the effects

of linear dynamic coupling terms by solving a trial problem and concluded that these terms produced significant changes in the predicted response at all of the frequencies associated with tooth passing. In all of these studies, the transfer matrix method was employed by developing the method for nonlinear systems whenever it was necessary. Blanding [142] also used the transfer matrix method in his dissertation published in 1985. The main emphasis in this work was placed on the derivation of the time varying stiffness tensor representing the involute spur gear mesh. The effects considered in the stiffness derivation were bending, shear, compression and local contact deformation. In the model developed by Buckens [143] in 1980 several simplifying assumptions are made but the elasticity and damping of the bearings as well as those of the shafts, and the damping due to friction at the contact between the gear teeth are retained. In his model it was assumed that the contact between the gears is never interrupted. Iida et al. [144-147] have published a series of papers between 1980 and 1986 on the coupled torsional-transverse vibrations of geared rotors. Transverse vibrations in tooth sliding and power transmitting directions were considered and it was shown that transverse vibrations couple with torsional vibrations even though gyroscopic effects are neglected. Ignoring the compliance ofthe gear tooth and other non-linear effects resulted in a linear model which was used to determine the natural frequencies and mode shapes. In their early work [144], a two shaft - two gear system was analyzed by assuming that one of the shafts was rigid, and the response to gear eccentricity and mass unbalance was determined. In later papers, a two shaft - four gear system was modeled by considering the torsional flexibilities of all shafts, but the transverse flexibility of only one shaft. In their recent paper [147], however, all shafts were assumed rigid in the transverse direction but the countershaft was assumed to be softly supported.
In 1981, Hagiwara, Ida and Kikuchi [148] used a simple model to study the vibration of geared shafts due to unbalanced and run-out errors. The lateral flexibilities of shafts were considered using discrete stillness values. Journal bearings were represented by damping and stillness matrices of order two which were calculated from Reynolds equation as a function of constant tooth force, rotating speed, clearance and oil viscosity. A constant mesh stiffness was assumed and the backlash and tooth separation were not considered in the analysis. It was both analytically and experimentally observed that unbalance forces and gear errors can excite both torsional and lateral modes, and large displacements can be observed in torsional modes. Iwatsubo, Arii and Kawai [149] studied the rotor dynamics problem of geared shafts by including a constant mesh stillness and the forcing due to unbalanced mass but by neglecting the tooth profile error and backlash. The transfer matrix method was employed in the solution and free and forced vibration analyses were made. In a subsequent paper [150] the authors solved a similar problem by including the effects of periodic variation of tooth mesh stillness and a tooth profile correction. In this study a stability analysis was also made by assuming a rectangular mesh stiffness variation. Neriya, Bhat and Sankar[151, 152] found the finite element formulation very useful in the dynamic analysis of geared trained rotors, since the coupling action in the gear pairs could be easily incorporated into the mass and stillness matrices. They modeled a single gear as a t~vo mass - two spring - two damper system, one of the set representing a tooth and the other the gear itself. In their earlier work [153] the shafts were assumed to be massless and an equivalent discrete model including lateral and torsional stillness of shafts \vas used. In the later studies [151,152], the shafts in the system were modeled by finite elements, and the coupling action between torsion and flexure was introduced in the model at the pair locations. A constant mesh stillness was assumed and the natural frequencies of the resulting linear system were obtained. The response of the system to mass unbalance and to geometric eccentricity in the gear, and the resulting dynamic tooth load were

calculated by using undamped modes of the system and equivalent modal damping values. Several numerical results were presented and discussed.

OTHER MATHEMATICAL MODELS

Another extreme in the dynamic modelling of gears is to neglect the flexibility of gear teeth and to consider the problem as a torsional vibration problem. A model for such an analysis consists of torsional springs representing the torsional flexibility of gear-carrying shafts, and rigid disks representing the interia of gears and shafts. Although such models were generally used to determine the natural frequencies of multi-gear-shaft systems [154-156], some investigators have used the rigid gear tooth assumption even in determining dynamic loads or the effect of gear errors upon the dynamic behavior of the system. For instance, in 1968 Rieger [157,158] modelled a drive train for torsional vibrations by assuming rigid gears, and studied the ellect of various types of gear errors. In this model, even the inertia of each gear was neglected. Mahalingam and Bishop [159] used modal analysis in the solution of a torsional model of a pair of gears. The response of the system to a displacement excitation representing periodic or transient static transmission error was calculated. Radzimovsky and Mirarefi [160] modelled a gear testing machine for torsional vibrations by assuming rigid gear teeth. They studied the effects ofseveral factors on the efficiency of gear drives and the coefficient of friction. Ikeda and Muto [161] studied the vibrations of a gear pair due to transmission errors and tooth frictions by again using the rigid gear tooth assumption. Their single degree of freedom model included gear inertias, torsional flexibility of the shaft, and damping. The calculated gear vibrations compared well with the experimental values in the frequency range tested. Also in the models of Wang [162,163] rigid gear teeth were assumed. He developed two models [162]: a two mass model without an elastic element and a three mass - one spring system. In both models time varying backlash, impact and displacement excitations were considered, and dynamic loads due to backlash impact were calculated by using a piecewise linear iteration technique. The theoretical predictions were experimentally verified [162,163] and it was shown that severe tooth loads may occur in lightly loaded gears due to impact. In another paper, Wang [164] derived Hertzian impact formulas for a crossed helical gear pair as an example· for Hertzian impact loads arising in rotational systems with backlash. These studies and his models, which cannot easily be grouped according to the classification made here lead to his interesting model which was discussed previously [25]. Another example for a mathematical model with rigid teeth assumption to calculate dynamic gear tooth load is the model of Osman, Bahgat and Sankar[l65, 166]. They studied the effect of bearing clearances on the dynamic response and dynamic tooth loads of spur gears and reached the conclusion that bearing clearances have considerable effects on the dynamics of gears, especially at high speed. Their analysis basiCally relies on the geometric computation of some angles and the use of rigid body dynamics.

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