AN IMPENDANT METHOD OF DETECTING A REDUCTION IN HYDRAULIC RESISTANCE IN ARTERIAL VESSELS

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Abstract: *The article deals with the problem of stationary blood flow in vessels with permeable walls. To determine the hydraulic resistance in an arterial vessel, the blood is considered to be a Newtonian viscous fluid, and the flow is stationary. In solving problems, formulas are obtained for determining the corresponding hydrodynamic parameters, such as velocity, fluid flow, and pressure gradient. Impedance method determined the hydraulic resistance. With a steady flow, the hydraulic resistance in the permeable vessel depends significantly on the permeability coefficient: with increasing this coefficient, it decreases.*

Keywords: *newton's liquid, continuity, oscillation parameter, pulsating motions*

Introduction

During the contraction of the heart from the heart along the walls of the blood vessels, a wave of pressure propagates, which is called a pulse wave. As the wave moves away from the heart, this wave gradually weakens and practically fades in the capillaries. The speed of the pulse wave propagation depends on many factors, among which, for example, the elastic-viscous properties of the vessel wall, blood pressure, its density, viscosity, permeability of the vessel wall, etc. can be noted. In most works,[1-6] the main attention is paid to determining the propagation of a pulse pressure wave taking into account the elasticity of the vessel wall, and its permeability is not taken into account anywhere. However, the permeability of the wall significantly affects the propagation of the pulse pressure wave and its damping. The branching of the arterial tree is modelled by introducing a "permeability" of the wall with volume outflows of transverse velocity to the walls. In fact, this velocity is a strongly discontinuous function of the coordinate, when using the model of a permeable tube this function is always smoothed by the wall surface. Apparently, the idea of modelling an artery in the form of a permeable tube was first expressed in[7] and is developed further in.[8] The most complete study of such a model is contained in,[9] where the system of blood circulation of a dog is modelled. Unlike other works,[2-6] to determine the hydraulic resistance in the central arterial vessel, the formulation of the problem of pulsating blood flow in the arterial channel was considered in,[10,11] where the outflow of blood is mathematically modelled as a permeable wall of blood vessels. It should be noted that in addition to wave flows, it is advisable to study the steady flow of blood in a vessel with permeable walls. However, this case does not follow from the solution for wave fluxes as the limit as the frequency tends to zero, so the stationary problem should be considered in a separate formulation.

$$\begin{cases} \dfrac{\partial \vartheta_x}{\partial t} = 0, \quad \dfrac{\partial \vartheta_r}{\partial t} = 0, \\ \dfrac{\partial u_x}{\partial t} = \dfrac{\partial u_r}{\partial t} = 0. \end{cases}$$

Taking them into account, we obtain from [1-5] a system of equations

$$\begin{cases} \dfrac{1}{\rho}\dfrac{\partial p}{\partial x} = \vartheta\left(\dfrac{\partial^2 \vartheta_x}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \vartheta_x}{\partial r} + \dfrac{\partial^2 \vartheta_x}{\partial x^2}\right), \\ \dfrac{1}{\rho}\dfrac{\partial p}{\partial r} = \vartheta\left(\dfrac{\partial^2 \vartheta_r}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \vartheta_r}{\partial r} - \dfrac{1}{r^2}\vartheta_r + \dfrac{\partial^2 \vartheta_r}{\partial x^2}\right), \\ \dfrac{\partial \vartheta_r}{\partial r} + \dfrac{1}{r}\vartheta_r + \dfrac{\partial \vartheta_x}{\partial x} = 0. \end{cases} \qquad (1)$$

And boundary conditions

$$\begin{cases} \vartheta_r = \dfrac{R\gamma^*}{\mu}\left(p - p_0\right), \ \vartheta_x = 0, \quad r = R, \\ \dfrac{\partial \vartheta_x}{\partial r} = 0, \ \vartheta_r = 0, \quad r = 0, \\ p = p_0, \ Q = Q_0, \quad x = 0. \end{cases} \qquad (2)$$

Where ρ-density of blood; r - radial; x - longitudinal coordinate; ϑ_r and ϑ_x - radial and axial velocity components; ϑ - cinematic viscosity of blood; p - internal pressure; p_0 -pressure in the environment; Q_0 - average pressure values and flow rate; γ^* - coefficient of permeability.

From the system (1), after making some calculations, we find the equation for the pressure

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial x^2} = 0 \qquad (3)$$

According to which in a stationary flow the pressure distribution obeys the Laplace equation. We seek the solution of (3) in the form

$$p = p_1(r)e^{i\gamma_n x} + p_2(r)e^{-i\gamma_n x} \qquad (4)$$

This procedure follows from the general solution of[3] under the condition that. In this case, the complex argument is transformed to the real one, i.e.

$$p = p_1(r)\ell^{\left(\frac{\lambda}{R}x\right)} + p_2(r)\ell^{\left(-\frac{\lambda}{R}x\right)} \qquad (5)$$

Here all functions are real. Therefore, in the stationary problem, there is no need for a complex solution.

The substitution of (5) into (3) leads to the Bessley equation

$$\frac{d^2 p_{12}}{dr^2} + \frac{1}{r}\frac{dp_{12}}{dr} + \frac{\lambda^2}{R^2} p_{12} = 0 \qquad (6)$$

His fundamental decisions will be:

$J_0\left(\frac{\lambda}{R}r\right)$ and $Y_0\left(\frac{\lambda}{R}r\right)$

In this case the general solution has the form

$$p_{12} = c_{12}J_0\left(\frac{\lambda}{R}r\right) + d_{12}Y_0\left(\frac{\lambda}{R}r\right)$$

At, the pressure becomes infinite, so the coefficients in front of the function $Y_0\left(\frac{\lambda}{R}r\right)$ must vanish and in this case the solution of (6) takes the form

$$p_{12} = c_{12}J_0\left(\frac{\lambda}{R}r\right) \qquad (7)$$

Or finally, we get a solution

$$p - p_0 = J_0\left(\frac{\lambda}{R}r\right)\left[c_1\ell^{\left(\frac{\lambda}{R}x\right)} + c_2\ell^{\left(-\frac{\lambda}{R}x\right)}\right] \qquad (8)$$

Where the coefficients c_1 and c_2 are determined from the boundary conditions (2). The calculation of (8) for the first equation of system (1) gives the equation

$$\frac{\partial^2 \vartheta_x}{\partial r^2} + \frac{1}{r}\frac{\partial \vartheta_x}{\partial x^2} = \frac{\lambda}{\mu R} J_0\left(\frac{\lambda}{R}r\right)\left[c_1\ell^{\left(\frac{\lambda}{R}x\right)} - c_2\ell^{\left(-\frac{\lambda}{R}x\right)}\right] \qquad (9)$$

whose solution is sought in the form

$$\vartheta_x = \vartheta_{1x}(r)\left[c_1\ell^{\left(\frac{\lambda}{R}x\right)} - c_2\ell^{\left(-\frac{\lambda}{R}x\right)}\right] \quad (10)$$

Then

$$\frac{d^2\vartheta_{1x}(r)}{dr^2} + \frac{1}{r}\frac{d\vartheta_{1x}(r)}{dr} + \frac{\lambda^2}{R^2}\vartheta_{1x} = \frac{\lambda}{\mu R}J_0\left(\frac{\lambda}{R}r\right) \quad (11)$$

Equation (11) is solved under the condition that the liquid adheres to the wall of the tube, i.e. ϑ_x at $r = R$ and limitations $\vartheta_x < \infty$ on the axis of the pipe

The fundamental solutions (11) of the Bessel function of zero order are expressed in this way

$$J_0\left(\frac{\lambda}{R}r\right) \text{ and } Y_0\left(\frac{\lambda}{R}r\right) \quad (12)$$

Equation (11) is inhomogeneous, and therefore its particular solution is found from formula

$$\vartheta_{1x}^* = \frac{\pi}{2}Y_0(x)\int xJ_0(x)f(x)dx - \frac{\pi}{2}J_0(x)\int xJ_0(x)f(x)dx \quad (13)$$

Where

$$f(x) = \frac{\lambda}{\mu R}J_0(x), \quad x\frac{\lambda}{R}r$$

Sometimes, taking into account the relation

$$J_0(x)Y_0'(x) - Y_0(x)J_0'(x) = \frac{2}{\pi x} \quad (14)$$

We get it

$$\vartheta_{1x}^* = \frac{1}{2\mu}rJ_1\left(\frac{\lambda}{R}r\right) \quad (15)$$

The use of the fundamental solution (12) and the limited velocity on the axis of the tube gives

$$\vartheta_{1x} = c_1J_0\left(\frac{\lambda}{R}r\right) + \frac{1}{2\mu}rJ_1\left(\frac{\lambda}{R}r\right) \quad (16)$$

From condition $\vartheta_x = 0$ at $r = R$, define

$$\overline{c}_1 = \frac{1}{2\mu}\frac{RJ_1(\lambda)}{J_0(\lambda)} \quad (17)$$

$$\vartheta_x = \frac{R}{2\mu}\left\{\frac{r}{R}J_1\left(\frac{\lambda}{R}r\right) - \frac{J_1(\lambda)}{J_0(\lambda)}J_0\left(\frac{\lambda}{R}r\right)\right\}X\left\{c_1\ell^{\left(\frac{\lambda}{R}x\right)} - c_2\ell^{\left(-\frac{\lambda}{R}x\right)}\right\} \quad (18)$$

In the same way, using the solution (18) and the second equation of the system (1), we obtain a formula for the distribution of the transverse velocity:

$$\vartheta_r = \frac{R}{2\mu\lambda}\left\{\frac{\lambda r}{R}J_0\left(\frac{\lambda}{R}r\right) + \left[\frac{\lambda J_1(\lambda)}{J_0(\lambda)} - 1\right]J_1\left(\frac{\lambda}{R}r\right)\right\}X\left\{c_1\ell^{\left(\frac{\lambda}{R}x\right)} - c_2\ell^{\left(-\frac{\lambda}{R}x\right)}\right\} \quad (19)$$

Using the wall permeability condition (2) leads to an equation for determining the eigenvalue λ

$$\gamma^* = \frac{1}{2\lambda}\left\{\lambda + \frac{J_1(\lambda)}{J_0(\lambda)}\left[\frac{\lambda J_1(\lambda)}{J_0(\lambda)} - 1\right]\right\} \quad (20)$$

If $J^* << I$ then, expanding the functions $J_0(\lambda),\ J_1(\lambda)$ in convergent series

$$\begin{cases} J_0(z) = 1 - \dfrac{\left(\frac{z}{2}\right)^2}{1} + \dfrac{\left(\frac{z}{2}\right)^4}{1^2 2^2} + \dfrac{\left(\frac{z}{2}\right)^6}{1^2 2^2 3^2} + \dots \\ J_1(z) = \dfrac{z}{2} - \dfrac{\left(\frac{z}{2}\right)^3}{1^2 2} + \dfrac{\left(\frac{z}{2}\right)^5}{1^2 2^2 3} + \dfrac{\left(\frac{z}{2}\right)^7}{1^2 2^2 3^2 4} + \dots \end{cases} \quad (21)$$

From (20), after some calculations, we find

$$\lambda = 4\sqrt{\gamma^*} \quad (22)$$

Equation (20) is transcendental and has an infinite set of roots λ_n Summation over eigenvalues λ_n gives formulas for pressure, radial and axial velocities

$$p - p_0 = \sum_{n=1}^{\infty} J_0\left(\frac{\lambda_n}{R} r\right)\left[A_n \ell^{\left(\frac{\lambda_n}{R}x\right)} + B_n \ell^{\left(-\frac{\lambda_n}{R}x\right)}\right]$$

$$\begin{cases} \vartheta_x = \sum_{n=1}^{\infty} \frac{R}{2\mu}\left\{\frac{r}{R} J_1\left(\frac{\lambda_n}{R} r\right) \frac{J_1(\lambda_n)}{J_0(\lambda_n)} J_0\left(\frac{\lambda_n}{R} r\right)\right\} X \left\{A_n \ell^{\left(\frac{\lambda_n}{R}x\right)} - B_n \ell^{\left(-\frac{\lambda_n}{R}x\right)}\right\} \\ \vartheta_r = \sum_{n=1}^{\infty} \frac{R}{2\mu\lambda_n}\left\{\frac{\lambda_n r}{R} J_0\left(\frac{\lambda_n}{R} r\right) + \left[\frac{\lambda_n J_1(\lambda_n)}{J_0(\lambda_n)} - 1\right] J_0\left(\frac{\lambda_n}{R} r\right)\right\} X \left\{A_n \ell^{\left(\frac{\lambda_n}{R}x\right)} + B_n \ell^{\left(-\frac{\lambda_n}{R}x\right)}\right\} \end{cases} \quad (23)$$

Where

$$\begin{cases} A_n = \frac{\lambda_n(\bar{p} - p_0)}{4J_1(\lambda)} - \frac{\frac{\mu\lambda_n^2 Q_0}{\pi R^3}}{2\left(\left(\lambda_n J_0(\lambda_n) + J_1(\lambda_n)\right)\left[\frac{\lambda_n J_1(\lambda_n)}{J_0(\lambda_n)} - 1\right]\right)} \\ B_n = \frac{\lambda_n(\bar{p} - p_0)}{4J_1(\lambda_n)} - \frac{\frac{\mu\lambda_n^2 Q_0}{\pi R^3}}{2\left(\left(\lambda_n J_0(\lambda_n) + J_1(\lambda_n)\right)\left[\frac{\lambda_n J_1(\lambda_n)}{J_0(\lambda_n)} - 1\right]\right)} \end{cases} \quad (24)$$

Formulas (23) allow us to investigate the characteristics of the internal structure of the flow of viscous blood in a pipe with permeable walls: hydrodynamic resistance on a permeable wall, fluid flow along the length of the tube, and a number of other hydrodynamic parameters. Using (23), we find the distribution of pressure drop, blood flow and shear stress on the wall:

$$-\frac{\partial p}{\partial x} = -\sum_{n=1}^{\infty} J_0\left(\frac{\lambda_n}{R} r\right)\left[A_n \frac{\lambda_n}{R} \ell^{\left(\frac{\lambda_n}{R}x\right)} - \frac{\lambda_n}{R} B_n \ell^{\left(-\frac{\lambda_n}{R}x\right)}\right] \quad (25)$$

$$Q = 2\pi\int_0^R r\vartheta_x(r,x)dr = \frac{\pi R^3}{\mu}\sum_{n=1}^{\infty}\left\{\frac{1}{\lambda^2} J_0(\lambda_n) X \left\{\lambda + \frac{J_1(\lambda_n)}{J_0(\lambda_n)}\left[\frac{J_1(\lambda_n)\lambda_n}{J_0(\lambda_n)} - 1\right]\right\} * \right.$$
$$\left. * \left(B_n \ell^{\left(-\frac{\lambda_n}{R}x\right)} - A_n \ell^{\left(\frac{\lambda_n}{R}x\right)}\right)\right\} \quad (26)$$

$$\tau_{cp} = \frac{1}{L}\int_0^L \mu\left(\frac{\partial \vartheta_x}{\partial r} + \frac{\partial \vartheta_r}{\partial x}\right)dr,$$

$$dx = \sum_{n=1}^{\infty}\left\{\frac{R}{L\lambda_n} J_0(\lambda_n)\left(2\lambda_n\gamma^* + \frac{J_1(\lambda_n)}{J_0(\lambda_n)}\right)\left[A_n\ell^{\left(\frac{\lambda_n}{R}L\right)} + B_n\ell^{\left(-\frac{\lambda_n}{R}L\right)} - (A_n + B_n)\right]\right\} \quad (27)$$

Using formulas (25) and (26), we determine the ratio of the pressure drop per unit length to the blood flow:

$$\frac{\left(-\frac{\partial \overline{p}}{\partial x}\right)}{Q} = \frac{8\mu}{\pi R^2}\sum_{n=1}^{\infty}\frac{\lambda_n}{8\gamma^*}\frac{J_1(\lambda_n)}{J_0(\lambda_n)} \quad (28)$$

Or, in a dimensionless form,

$$\frac{\pi R^4}{8\mu}\frac{\left(-\frac{\partial \overline{p}}{\partial x}\right)}{Q} = \frac{8\mu}{\pi R^2}\sum_{n=1}^{\infty}\frac{\lambda_n}{8\gamma^*}\frac{J_1(\lambda_n)}{J_0(\lambda_n)} \quad (29)$$

Attitude $\frac{\left(-\frac{\partial \overline{p}}{\partial x}\right)}{Q}$ is called the effective impedance, in the general case it is expressed by a function of a complex variable. Therefore, its real part determines the hydraulic resistance of the flow. It can be seen from Figure 1 that the effective impedance at a stationary flow in a permeable pipe essentially depends on the permeability coefficient: with an increase in this coefficient, it decreases. It can be seen from Figure 1 that in the arterial vessel the hydraulic resistance decreases due to the permeability of the wall, when it is 0.015, corresponds to a decrease by 9-10 percent.

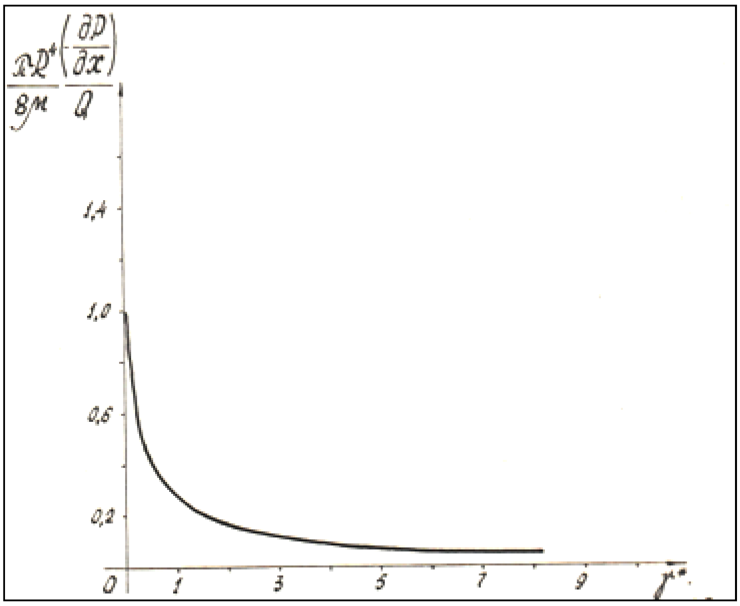

Figure 1 Change in the total resistance of blood in the arterial bed, depending on the permeability of the wall

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